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On a large time behavior of a solution to a one-dimensional free boundary problem for adsorption phenomena

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ABSTRACT

In this paper we consider a one-dimensional free boundary problem describing adsorption phenomena in a porous media. This problem was already proposed and we established a local existence in time and a uniqueness result in [1,4]. Here, through the discussion for the dynamics of the free boundary in wetting and drying processes we suppose a specific form of the growth rate condition for the free boundary. By using this form in our problem we can obtain the global existence of a solution in time and convergence of the free boundary as $t \to \infty$.

1. Introduction

We study a free boundary problem proposed as a mathematical model for a drying and wetting process in a porous media in our previous papers [1,4]. In this model we consider the processes in one hole of the media and regard the hole as a one-dimensional interval $[0, L]$, where L is the length of the hole. Also, we suppose that the interval consists of the water-drop (liquid) zone $[0, s(t))$ and the air zone $(s(t), L]$, and denote by u the relative humidity in the air zone, where $t \in [0, T]$ is the time variable and s is a curve with $0 < s < T$ on $[0, T]$ for $T > 0$ (see Fig. 1). The problem is to find a pair of s and the function u on $Q_s(T) := \{(t, x); 0 < t < T, s(t) < x < L\}$ (see Fig. 2) satisfying

$$\rho_g u_t - \kappa u_{xx} = 0 \text{ in } Q_s(T), \tag{1.1}$$

$$u(t, L) = b(t) \text{ for } 0 < t < T, \tag{1.2}$$

$$\dot{s}(t)(:= \frac{d}{dt}s(t)) = \alpha(s(t), u(t, s(t))) \text{ for } 0 < t < T, \tag{1.3}$$

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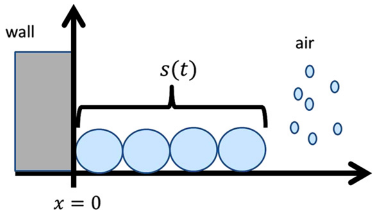

Fig. 1. Image of one hole.

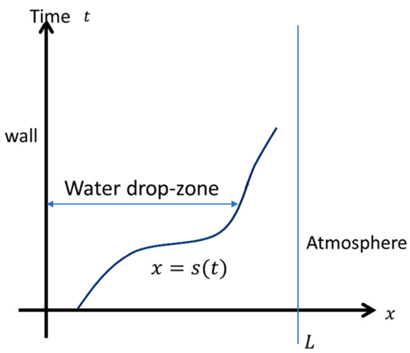

Fig. 2. Domain of the problem.

$$\kappa u_x(t, s(t)) = (\rho_a - \rho_g u(t, s(t)))\dot{s}(t) \text{ for } 0 < t < T, \tag{1.4}$$

$$s(0) = s_0, \tag{1.5}$$

$$u(x, 0) = u_0(x) \text{ for } s_0 < x < L, \tag{1.6}$$

where ρ_a and ρ_g are constants of the density of the aqueous-H_2O and the gaseous-H_2O, respectively, κ is a diffusion constant of the gaseous-H_2O, a continuous function α on $\mathbb{R}^2$ indicates the growth rate of the liquid zone, b is a given boundary function on $[0, T]$, and s_0 and u_0 are initial data.

In [1,4] under some conditions for α, b, s_0 and u_0 we have proved that the above problem (1.1)–(1.6) has a solution $\{s, u\}$ on $[0, T_0]$ for some $0 < T_0 \leq T$ and admits at most one solution.

Our aim of this paper is to show a large time behavior result for the solution of the problem. For the analysis to the large time behavior we need some uniform estimates for solutions with respect to t. In order to get these estimates we assume more conditions for α. For the following physical iterpretaion we refer to [2,3].

First, we suppose that the grow rate α is equals to the difference of the rates r_1 and r_2, that is, $\alpha = r_1 - r_2$, where r_1 and r_2 are the rate from moisture in air to water-droplet and from water-droplet to moisture on the free boundary. By collision of moisture to the water droplet or the wall the moisture becomes water-droplet. Then the rate r_1 is proportionally to the density of the gaseous-H_2O near the free boundary. This leads to $r_1 = au(t, s(t))$, where a is a positive constant. Next, we consider the behavior of the water droplet near the free boundary (see Fig. 1). Let f_1 and f_2 be attractive forces between the water droplet and the wall, and between water droplets, respectively. If the sum of the forces f_1 and f_2 is weak, then the water droplet becomes moisture, easily. Otherwise, the water droplet stays there. Thus we may suppose that r_2 is a non-increasing function of $f_1 + f_2$. Moreover, we assume that $r_2 \to 0$ as $f_1 + f_2 \to \infty$. Since by elementary physics the force f_1 is given by the form $f_1 = cs^{-m}$, where c and m are positive constants and we can regard that f_2 is a positive constant, we suppose

$$r_2 = \frac{a}{c_1(f_1 + f_2)} = \frac{a}{c_1(cs^{-m} + f_2)},$$