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Optimal decay rates and asymptotic profile for the plate equation with structural damping

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ABSTRACT

In this work we study decay rates for the L^2-norm of solutions to the plate equation with fractional damping and a pseudo fractional rotational inertia term. We also show that the decay rates depending on the fractional power of the damping term are optimal using an asymptotic expansion of the corresponding solution of the related equation in the Fourier space.

1. Introduction

We consider in this work the following Cauchy problem for the plate equation with a fractional damping and a pseudo fractional rotational inertia term in $\mathbb{R}^n$ $(n \geq 2)$:

$$\begin{cases} u_{tt}(t,x) + (-\Delta)^{\delta} u_{tt}(t,x) + \alpha \Delta^2 u(t,x) - \Delta u(t,x) + (-\Delta)^{\theta} u_t(t,x) = 0, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x) \end{cases} \tag{1}$$

with $(t,x) \in (0,\infty) \times \mathbb{R}^n$, $\alpha > 0$, θ and δ in $[0,\infty)$ and the initial date satisfying

$$[u_0, u_1] \in \Big(H^{4-\delta}(\mathbb{R}^n) \cap L^1(\mathbb{R}^n) \Big) \times \Big(H^2(\mathbb{R}^n) \cap L^{1,\gamma}(\mathbb{R}^n) \cap L^1(\mathbb{R}^n) \Big), \quad \gamma \in (0, \min\{1, \delta\}].$$

Some fourth-order partial differential equations arise in problems of solid mechanics. In particular, evolution partial differential equations of fourth-order appear in the theory of thin plates and beams. Models to

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study the vibrations of thin plates ($n = 2$) given by the full von Kármán system have been studied by several authors, in particular by Puel–Tucsnak [22], Ciarlet [3], Lasiecka–Benabdallah [16] and Koch–Lasiecka [15]. Perla Menzala–Zuazua [20] studied the full von Kármán system and they showed that the Timoshenko's model

$$u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u + u = 0, \qquad \text{in } \mathbb{R}^2 \times (0, \infty) \tag{2}$$

may be obtained as limit of a full von Kármán system when suitable parameters go to zero. The term $-\gamma\Delta u_{tt}$ in plate equation (2) is to absorbed in the model the rotational inertia effects at the point x of the plate in a positive time t. It is well known that the plate equation (2) with such term is a hyperbolic equation with finite speed of propagation, whereas the plate model (1) for the case $\delta = 0$ and $\theta = 1$ or 2 has the property of infinite speed of propagation. Moreover, as far as we know, the classification of the plate model (1) for $\delta \in (0, 1)$ is still open even for $\theta = 0$ or $\theta = 1$. We conjecture that for δ near $\delta = 1$ the equation continues to be hyperbolic.

A more general model to study vibrations of a thin plate is given by

$$u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u + g_0(u_t) - \operatorname{div} g_1(\nabla u_t) = 0. \tag{3}$$

Such model has been studied by several authors as [9,8,2,23] and, in particular, by Sugitani–Kawashima [25] which considered in $\mathbb{R}^n$ the cases $g_1 = 0$ and $g_0 = \mathrm{Id} - f$. Furthermore, there are some papers in which a strong damping of type $(-\Delta)^2 u_t$ is considered in the model (3), in place of the damping given in $g_0(u_t) - \operatorname{div} g_1(\nabla u_t)$ (see, e.g. [27,18,28] and references therein).

Problems of type (1) with $\delta = 0, 1$ have been extensively studied for the cases $\theta = 0,\ 1,\ 2$. Recently several authors studied evolution equations with fractional Laplacian operator $(-\Delta)^\theta$. For the plate equation we mention the works by Ikehata–Soga [13], Charão–da Luz–Ikehata [5] and Astaburuaga–Fernandez–Menzala [1] that studied the dynamical von Kármán equations in the presence of fractional dissipation. Therefore, it is very important from a mathematical point of view to study the plate equation with generalized rotational inertia term under effects of an intermediate damping as in our model (1) with $\delta \geq 0$ and $\theta > 0$. In particular, the cases $\theta = 0,\ 1/2,\ 1,\ 2$ combined with the cases $\delta = 0, 1$ also have important physical motivation. Moreover, in solid mechanic the spatial derivatives of the vector displacement define the components of the strain tensor and the time derivative of the strain is a strain rate, that is, it works as a dissipation on the model. For example, in dimension $n = 1$ the rotational term $-\Delta u_{tt}$ models small rotations of the transversal sections of the beam. We also note that the hyperbolic type model (1) with $\delta > 0$ is more complicated to be investigated than the non-hyperbolic one (the case $\delta = 0$). Due to this, it is necessary to impose additional regularity on the initial data to control the vibrations of the beam and the rotations of such sections in the case $\theta \in [0, 1)$. Due to the strong damping given in the case $\theta = 1$ no additional regularity on the initial data is necessary in this case. The same occurs for the plate equation case (dimension $n = 2$). Thus, for example, from the engineering point of view, time spatial derivatives of the vector velocity are very important to control dissipations of models of vibrations, even in the case of fractional spatial derivatives. Fractional spatial derivatives also can be introduced to control weak rotational inertia effects in the plate as in our model (1) in the case $0 < \delta < 1$. Finally, we note that in previous works authors studied fractional damping only in the case $\theta \in [0, 1]$ and in this paper we also consider cases with $\theta > 1$.

We mention several related works about (1). In the case when $\delta = \alpha = 0$ and general $\theta \in [0, 1]$ (that is, the damped wave equation case), the corresponding Cauchy problem is studied by Ikehata–Natsume [12], and there they obtained precise decay estimates of the total energy and L^2-norm for solutions based on the energy method in the Fourier space plus special one due to [17] and the references therein. An improvement of the results of [12] was given in Charão–da Luz–Ikehata [4] by introducing a new energy method in the Fourier space. In the case of $\delta = \alpha = 0$ and general θ, to begin with, one has to cite three important papers