J. Math. Anal. Appl. 440 (2016) 750–772

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

# Operator theoretic framework for optimal placement of sensors and actuators for control of nonequilibrium dynamics

S. Sinha[a], U. Vaidya[a,*], R. Rajaram[b]

[a] Department of Electrical and Computer Engineering, Iowa State University, Ames, IA 50011, United States
[b] Department of Mathematical Sciences, Kent State University, Ashtabula, OH 44004, United States

ARTICLE INFO

*Article history:*
Received 21 February 2014
Available online 24 March 2016
Submitted by Goong Chen

*Keywords:*
Dynamical system
Control of dynamical systems
Optimization
Nonequilibrium dynamics
Operator theory

ABSTRACT

In this paper, we present a novel operator theoretic framework for optimal placement of actuators and sensors in nonlinear systems. The problem is motivated by its application to control of nonequilibrium dynamics in the form of temperature in building systems and control of oil spill in oceanographic flow. The controlled evolution of a passive scalar field, modeling the temperature distribution or density of oil dispersant, is governed by a linear advection partial differential equation (PDE) with spatially located actuators and sensors. Spatial locations of actuators and sensors are optimized to maximize the controllability and observability regions of the linear advection PDE. Linear transfer Perron–Frobenius and Koopman operators, associated with the advective velocity field, are used to provide an analytical characterization for the controllable and observable spaces of the advection PDE. Set-oriented numerical methods are proposed for the finite dimensional approximation of the linear transfer operators. The finite dimensional approximation is shown to introduce weaker notion of controllability and observability, referred to as coarse controllability and observability. The finite dimensional approximation is used to formulate the optimization problem for the optimal placement of sensors and actuators. The optimal placement problem is a combinatorial optimization problem. However, the positivity property of the linear transfer operator is exploited to provide an exact solution to the optimal placement problem using greedy algorithm. Application of the framework is demonstrated for the placement of sensors in a building system for the detection of contaminants and for optimal release of dispersant location for control of contaminant in a Double Gyre velocity field. Simulation results reveal interesting connections between the optimal location of actuators and sensors, maximizing the controllability and observability regions respectively, and the coherent structures in the fluid flow.

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* Corresponding author.
*E-mail address:* ugvaidya@iastate.edu (U. Vaidya).

http://dx.doi.org/10.1016/j.jmaa.2016.03.058
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## 1. Introduction

We study the problem of optimal placement of actuators and sensors for the control of nonequilibrium dynamics. The problem is motivated by the control of nonequilibrium dynamics as seen in systems involving fluid flow. For example, in building systems applications, the objective is to determine optimal locations of actuators in the form of vents/ducts for the control of temperature [30,1]. For the control of contaminants from an oil spill in oceanographic flows, the goal is to determine the optimal location for the release of dispersant. The distribution of temperature in the room or the dispersant on the ocean surface is assumed to be modeled by a passive scalar density. The passive scalar is advected by the fluid flow velocity field. Under some simplifying assumptions of negligible buoyancy and diffusion, the evolution of the passive scalar density is modeled using the linear advection partial differential equation (PDE). For the purpose of control and observation, we assume that the actuators and sensors are spatially located.

The problem of control of temperature in a building system or prediction of contaminants in fluid flow is challenging because of the nonequilibrium nature of dynamics that are involved. The fluid flow velocity fields involved in these applications are quite complex with dynamics consisting of multiple equilibrium points, periodic orbits, limit cycles and chaotic attractors [23,18]. Although the point-wise evolution of a particle under the influence of the velocity field is complex, the evolution of density, modeling the ensemble behavior, is linear and is described by the linear advection partial differential equation (PDE). In this paper, we have developed a systematic approach involving the linear advection PDE for optimal placement of actuators and sensors for the control of nonequilibrium dynamics. There is an extensive amount of literature on actuator and sensor placement for a linear PDE with [15] providing an excellent review of the results. Most of the methods for the optimization of sensors and actuators location involve finite dimensional approximation of the infinite dimensional system. In [6], sensor and actuator placement for diffusive and heat type partial differential equations are discussed. The placement problem for flexible structures is studied in [10]. In [30], we provided analytical expression for the finite time controllability and observability gramian for the advection PDE. Selection criteria for the optimal location of actuators and sensors were proposed based on the maximization of gramians. In this paper, we provide a systematic linear programming-based approach for the optimal placement of actuators and sensors. This linear programming formalism is made possible because of the analytical characterization of infinite time controllability and observability gramians. Characterization of the infinite time gramians is provided in terms of *Lyapunov density* [24,29,27], which was introduced to verify a weaker notion of almost everywhere stability for nonlinear systems. The Lyapunov density-based gramian construction provides for the characterization of controllable and observable regions over infinite time with spatially located actuators and sensors respectively. Gramian construction for almost everywhere stable nonlinear systems using operator theoretic method has been proposed in [28], however one of the main highlights of this result is that the infinite time characterization of the gramian is made possible even for systems with complex dynamics with possible multiple steady states.

Following are the main contributions of this paper. In this paper, we provide an analytical characterization for the infinite time controllability and observability gramians using a linear transfer operator framework. Set-oriented numerical methods are proposed for the finite dimensional approximation of the gramian operators and hence for the approximation of the controllable and observable regions in the physical space. We show that the set-oriented numerical methods used for approximation introduce weaker notions of *coarse* controllability and observability, which roughly imply controllability and observability modulo the size of the cell used in the approximation. The finite dimensional approximation of the gramians are used in the formulation of the optimization problems for the optimal placement of sensors and actuators. The optimal placement problems are typically formulated as combinatorial optimization problem and are known to be NP-hard [20]. However, we exploit the positivity of the linear transfer operator to provide an exact solution to the optimal placement problem. In particular, we prove that a greedy algorithm will lead to an optimal solution to the placement problem. The application of the developed framework is demonstrated on the

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