J. Math. Anal. Appl. 440 (2016) 773–793

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

A mathematical model for forest growth dynamics

Lorenzo Fusi [a,*], Mario Primicerio [a], Atsushi Yagi [b]

[a] Università degli Studi di Firenze, Dipartimento di Matematica e Informatica "U. Dini", Viale Morgagni 67/a, 50134 Firenze, Italy
[b] Osaka University, Department of Applied Physics, Suita, Osaka 565-0871, Japan

ARTICLE INFO

Article history:
Received 11 January 2016
Available online 31 March 2016
Submitted by B. Kaltenbacher

Keywords:
Forest growth
ODEs
Stationary solutions

ABSTRACT

In this paper we consider a one dimensional mathematical model for a mono-species forest growth in an infinite domain. Following [15] we assume that the system can be described by the evolution of the young and old tree density and by the seed concentration. The mathematical formulation consists in a system of ODE's which we study analytically proving global existence and uniqueness. We discuss in some detail the stationary solutions and we perform some numerical simulations to illustrate the behavior of the solution.

1. Introduction

Conservation of forest resources is one of the most challenging problems in ecology and environmental science. The knowledge of forest dynamics is of crucial importance for maintaining the ecological integrity of forest ecosystem and for preserving biodiversity.

The fundamental issue in these problems is to predict the variation of tree density and age caused by internal and external factors. Various mathematical models have been proposed in the last decades. These works encompass individual-based models in a specific forest plot [3], individual-based space continuous models [10,9] and age-structured continuous space models [1,2,8].

The latter are of particular interest, as they take into account seed dispersion mechanisms and age dependent tree relationships. If we consider a mono-species forest, then the local dynamics can be described using a division of trees in two age classes, as in [8]. In this case the ecosystem is modeled describing the evolution of the "young" and "old" trees and assuming that the regeneration mechanism is governed by seed production, diffusion and establishment rate.

In [4–6] the authors solved the mono-species two-age classes problem in a 2D bounded domain with homogeneous Neumann boundary conditions. In particular they proved the existence of global solutions

* Corresponding author. Fax: +39 55 2751452.
E-mail address: fusi@math.unifi.it (L. Fusi).

http://dx.doi.org/10.1016/j.jmaa.2016.03.077

and investigated stability and instability for homogeneous stationary solutions. In [11–13] the same problem is considered for Dirichlet boundary conditions. In [14] the problem of the forest boundary is discussed. In [15] a modified model equation is introduced to get stationary solutions with compact support.

In this paper we consider the 1D version of the model presented in [15], assuming that the spatial domain of the problem is unbounded and that the seed concentration vanishes at infinity. Following [15] we initially consider a model in which the seed production rate is proportional to density of old trees. Under this assumption we determine a unique stationary solution with compact support and we perform some numerical simulations showing that such a solution is unstable. In particular we show that the model admits solutions in which the growth of the forest is unbounded, a clearly unrealistic result.

Motivated by this fact, we modify the model assuming that the effective seed production rate due to old trees is actually bounded, in the sense that it reaches a sort of "saturation" level when the density of old trees exceeds a certain value (crowding effect). The modified problem provides a unique bounded stationary solution with compact support and the system now does not admit unbounded solutions, as shown in numerical simulations.

2. The mathematical model

Following [15], we consider the one dimensional initial-boundary value problem

$$\begin{cases} u_t = \beta\delta(w - w^*)_+ - \gamma u - fu, \\ v_t = fu - hv, \\ w_t = dw_{xx} - \beta w + \alpha v_+ \end{cases} \tag{1}$$

with $x \in \mathbb{R}$ and $t > 0$. In (1) $u(x,t)$ denotes the density of young trees, $v(x,t)$ the density of old trees and $w(x,t)$ the density of seeds. The density $w^* > 0$ represents a threshold for the seeds concentration below which birth of new trees does not occur (the sign + stands for the positive part). The dimensions of u, v, w are the inverse of a length. The set of equations (1) describes the evolution of the system with

- α seed production rate;
- β seed deposition rate;
- γ mortality rate of young trees (this may depend on the concentration of old trees);
- $\delta \in (0,1]$ seed establishment parameter;
- f aging rate of young trees;
- h mortality rate of old trees;
- d diffusivity of seeds.

Problem (1) must be coupled with the initial condition for u, v, w

$$u(x,0) = u_o(x) \geq 0, \qquad v(x,0) = v_o(x) \geq 0, \tag{2}$$

$$w(x,0) = w_o(x) \geq 0, \quad x \in \mathbb{R}, \tag{3}$$

with

$$0 < \text{supp} \quad (u_o + v_0) = L < \infty,$$

and with the boundary conditions for w

$$0 \leqslant w(\pm\infty, t) = W, \qquad t \geq 0. \tag{4}$$