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Existence and energy decay rates of solutions to the variable-coefficient Euler–Bernoulli plate with a delay in localized nonlinear internal feedback ☆

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ABSTRACT

This paper is concerned with the existence and energy decay estimates of solutions to the variable-coefficient Euler–Bernoulli plate with a delay in localized nonlinear internal feedback. The existence of solutions to the nonlinear system is obtained by using Galerkin method combined with some energy estimates. Furthermore, applying Riemannian geometry method, we establish uniform decay rates of the plate equation with variable coefficients and a nonlinear delay term.

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1. Introduction

The main purpose of this paper is to study the existence and decay estimates for the energy of solutions to a variable-coefficient Euler–Bernoulli plate equation with nonlinear internal damping terms and in the presence of a nonlinear degenerate delay term. Here, the damping terms are localized in a neighborhood of a suitable portion of the boundary of the domain under consideration.

Let Ω be a bounded domain in $\mathbb{R}^n$ $(n \geq 2)$ with smooth boundary Γ, and consider the following damped plate equation with variable coefficients

$$\begin{cases} u_{tt} + \mathcal{A}^2 u + b(x)[\mu_1 \beta(u_t(x,t)) + \mu_2 \varphi(u_t(x,t-\tau))] = 0 & \text{in} \quad \Omega \times (0,\infty), \\ u = \dfrac{\partial u}{\partial \nu_A} = 0 & \text{on} \quad \Gamma \times (0,\infty), \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) & \text{in} \quad \Omega, \\ u_t(x,t-\tau) = k_0(x,t-\tau), & \text{in} \quad \Omega \times (0,\tau), \end{cases} \tag{1.1}$$

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where $\mathcal{A}u = \text{div}(A(x)\nabla u),\ u \in C_0^\infty(\mathbb{R}^n),\ x \in \mathbb{R}^n$, div X denotes the divergence of the vector field X in the Euclidean metric, and ∇u denotes the gradient of u in the Euclidean metric. $A(x) = (a_{ij}(x))$ is a matrix function with $a_{ij} = a_{ji}$ of C^∞ functions in $\mathbb{R}^n$, satisfying

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \lambda \sum_{i=1}^{n} \xi_i^2 \qquad \forall x \in \mathbb{R}^n, \quad 0 \neq \xi = (\xi_1, \xi_2 \cdots, \xi_n)^T \in \mathbb{R}^n, \tag{1.2}$$

for some positive constant λ. $\dfrac{\partial u}{\partial \nu_A} = \displaystyle\sum_{i,j=1}^{n} a_{ij}\frac{\partial u}{\partial x_j}\nu_i$, where $\nu = (\nu_1, \nu_2, \cdots, \nu_n)^T$ denotes the unit normal vector of the boundary Γ toward the exterior of Ω and $\nu_A = A\nu$. The localized function $b(x)$ is a nonnegative function which will be specified in Section 2. $\beta, \varphi : \mathbb{R} \to \mathbb{R}$ are given nonlinear functions. Here, $\tau > 0$ is a time delay, μ_1, μ_2 are real numbers with $\mu_1 > 0, \mu_2 \neq 0$, and the initial data (u_0, u_1, k_0) belongs to a suitable space.

In absence of delay ($\mu_2 = 0$), the model (1.1) is a plate equation with a localized damping. The analysis of stabilization when the damping is effective only on a subset of the domain Ω is much more subtle than that on the whole domain. Such problems have been extensively investigated in context of wave equations and literature on the subject is quite impressive (see for instance [10,13,18,27,39] and the references therein). The same problems have also been addressed for plate equations. But the corresponding results are not many and we refer the reader to [4,9,25,28–30].

Time delay is the property of a physical system by which the response to an applied force is delayed in its effect. Whenever material, information or energy is physically transmitted from one place to another, there is a delay associated with the transmission. Time delay so often arises in many physical, chemical, biological, and economical phenomena. These hereditary effects are sometime unavoidable, and they may turn a well-behaved system into a wild one. That is to say, they can induce some instabilities [5,6]. The stability issue of the systems with delay is, therefore, of theoretical and practical importance.

In recent years, stabilization of PDE with time-delay effects has become an active area of research [1,3,11,15,19–24,33]. There is a large number of literature on stabilization of the wave equation with a delay in the internal/boundary feedback (see for instance [1,3,11,15,19–23] and the references therein). However, to the knowledge of the authors, only a few papers [20,24,33] address stabilization of the plate equation with delay effects. Nicaise and Pignotti [20] obtained stability results for the Euler–Bernoulli plate equation with time delay regarding the plate equation as a particular case of more general systems. Yang [33] discussed the following Euler–Bernoulli viscoelastic equation with a delay

$$u_{tt}(x,t) + \triangle^2 u(x,t) - \int_0^t g(t-s)\triangle^2 u(x,s)ds + \mu_1 u_t(x,t) + \mu_2 u_t(x, t-\tau) = 0.$$

Under suitable assumptions on the coefficients $\mu_1,\ \mu_2$ and the relaxation function g, the existence and exponential decay rates of the energy for solutions to above problem are obtained. In [24], Park studied a weak viscoelastic equation but with a time-varying delay. By introducing suitable energy and Lyapunov functions, Park also proved exponential decay results of the energy for the concerned problem. Our aim in this work is to extend these existing decay results to a nonlinear plate equation with a delay and obtain general decay estimates for the energy of the nonlinear system (1.1) with a delay. For the research on stabilization of PDE with delay effects, even if in the case of wave equations, energy decay estimates are more considered in linear systems. It was not until the last few years that there was several literature on such subject considered in nonlinear systems [3,15]. In [15], the authors of this paper investigated a nonlinear wave equation with acoustic boundary conditions and a time-varying delay in the boundary feedback. By promoting the method applied with success to obtain energy decay estimates of a nonlinear system in [12]

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