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Equilibrium states of the Charney-DeVore quasi-geostrophic equation in mid-latitude atmosphere

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ABSTRACT

On the predictability of atmospheric blocking in the mid-latitude atmosphere, a quasi-geostrophic beta-plane channel model equation was introduced in a pioneering work by Charney and DeVore in 1979 based on the understanding of quasi-stationary weather patterns. The equation is driven by a zonal thermal flow representing a mid-latitude westerly jet and a wave function representing an ocean land topography. They truncated the equation into a three spectral mode model, which gives rise to the existence of multiple equilibrium states showing a blocking mechanism profile. Moreover, they predicted numerically the absence of the multiple equilibrium states with respect to the flat topography situation. In the present paper, this absence observation is discussed from rigorous analysis and the coexistence of two stable equilibrium states, similarly to the three mode model equilibrium states accounting for atmospheric blocking, is investigated numerically with the increment of topographic amplitude.

1. Introduction

The persistence of planetary scale atmospheric anomalies and their influence on weather patterns have long been noted by scientists. Atmospheric blocking in mid-latitude atmosphere, disturbing eastward motion process, is a typical example of a persistent anomaly effecting atmospheric circulation. For example, in an omega shape blocking pattern, a high-pressure ridge covering the centre of the omega block experiences dry weather while low pressure troughs on either side of the block are dominated by rain and clouds.

The investigation into the maintenance of atmospheric blocking episodes is of great importance in practical relevance for weather prediction and climate change simulations. In the celebrated work of Charney

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and DeVore [3], a β-plane atmospheric flow in the troposphere between a topography h and a free surface is governed by the conversation of potential vorticity

$$\partial_t \left(\Delta\psi - \frac{\psi}{\lambda^2} \right) + J(\psi, \Delta\psi + h + \beta y) + \kappa\Delta\psi = \tau \tag{1}$$

on a synoptic scale channel domain $[0, 2\pi L] \times [0, \pi L]$ for $L \sim 1000$ km. This equation was derived from a quasi-geostrophic approximation [2]. Here τ is a zonal thermal forcing, $J(\psi, \phi) = \partial_x\psi\partial_y\phi - \partial_y\psi\partial_x\phi$ is the Jacobian, λ is a parameter defined by the Coriolis force and gravity effect on the atmosphere, and the coefficient κ counts for the Ekman layers damping around the topography and the free surface.

The blocking events are explained as a quasi-stationary state. With nonlinear interactions of the zonal source and the Ekman damping, Charney and DeVore [3] suggested the coexistence of multiple equilibrium states of (1) due to topography disturbance around the zonal jet, which is a mean flow parallel to the circles of mid-latitude atmosphere.

In the study of Charney and DeVore [3], equation (1) is approximated by a three spectral mode truncation model as in the spectral mode truncation scheme of Lorenz [16]. Within certain parameter ranges, the three mode truncation model admits a high index, a low index and a medium index equilibrium states. The high index indicates the zonal flow component dominates stationary flow motion while the low index demonstrates the wave component related to h control the stationary fluid motion. The high index and the low index equilibrium states are stable and the medium index equilibrium state is unstable. Blocking occurs when the flow is in the vicinity of the low index equilibrium circulation. This heuristic study has been widely confirmed by climatologists (see, for example, Crommelin et al. [10], Legras and Ghil [15], Eert [11], Ierley and Sheremet [13], Jiang et al. [14], Pedlosky [17], Pierrehumbert and P. Malguzzi [18], Primeau [19], Rambaldi and Mo [20], Reinhold and Pierrehumbert [22], Tung and Rosenthal [24], Holloway and Yoden [27,28]).

Based on a finite difference scheme, numerical integrations of the barotropic atmospheric flow described by (1) were carried out on a 16×16 grid for the β-pane channel (see [3]). The existence of the multiple steady-state phenomenon was also confirmed by the grid-point scheme and the numerical integration flows are attracted by a low-index equilibrium states exhibiting atmospheric blocking (see [3, Figure 4]) when the non-dimensional topographic amplitude equals 0.2.

For the purely thermally driven flow without the topography involved, the multiple equilibria phenomenon was found in a six spectral mode truncation model but seems resultant from the truncation scheme as it disappears in the numerical integration via the grid-point model (see [3, Figure 5]). However, from view point of rigorous mathematical analysis, the understanding of equation (1) with respect to the equilibrium problem is missing, although [7,8] considered rigorous bifurcation problems on some fluid motion equations different to (1). Especially, the existence of multiple equilibrium states was examined by Chen [5] for purely thermally driven flow governed by (1) over a wider channel domain. The growth of the channel wideness increases the freedom of the fluid motion and hence complexifies the nonlinear behaviours of the circulation flow. The nonlinear complexity of (1) with a different thermally driven source was also recently discussed by Chen [6].

To address the importance of the bottom surface topography of the multiple equilibria technique given by Charney and DeVore [3], the purpose of the present study is two-fold. First, if the topographic wave is either flat or parallel to the zonal thermal flow, it is proved rigorously that the basic equilibrium state of (1) admits is globally stable with respect to the Fourier expansion perturbation of [3,4] in a Sobolev space. This result is stated as Theorem 3.1 in Section 3 and rules out the existence of the atmospheric blocking phenomenon. Second, for the topographically driven flow of [3] with the photographic amplitude increasing from 0 to 0.2, a quasi-spectral code is developed from [9,25,26] in Section 4 on numerical simulation of the losing global stability and the occurrence of multiple equilibrium states as described in the three mode truncation model of [3].