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Small perturbation of a surface: Full Maxwell's equations

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ABSTRACT

In this paper, we derive high-order terms in the asymptotic expansions of the boundary perturbations of the electromagnetic fields resulting from small perturbations of the shape of an inhomogeneity with C^d-boundary in $\mathbb{R}^d$, for $d = 2, 3$. For such solutions we provide a rigorous derivation that extends those already derived for small volume conductivity inhomogeneities. Our formulas may be expected to develop effective algorithms, aimed at determining certain properties of the shape of an object based on electromagnetic boundary measurements.

1. Introduction

In this paper we study the effect on solutions of Maxwell's equations of small perturbations of the shape of an inhomogeneity in the bounded medium. We derive asymptotic expansions (in principle, to high orders) for the resulting changes in the components of the electromagnetic field tangential to the boundary of the inhomogeneity. This work extends those previously derived in [4,9,16,18,23] to the case of surface perturbations, and is completely different from our investigations in [15].

Our proofs are based on nearly similar techniques but more elaborate arguments are needed.

To the best of our knowledge, this is the first work to rigorously investigate electromagnetic interface problem in two or three dimensional bounded domain and derive high-order terms in the asymptotic expansion of $(\nabla \times \mathbf{E}_\delta) \times \nu - (\nabla \times \mathbf{E}_0) \times \nu$ when $\delta \to 0$. In this paper, assuming that the unknown domain S_δ is a small perturbation of a sphere, we develop a relationship between generalized Fourier coefficients (in the vector spherical harmonic basis) of the perturbation of the shape and boundary measurements. Our formula may also be extended to those already derived for small volume inhomogeneities in [3,5–7,22] to develop effective algorithms for determining certain properties of the shape of an inhomogeneity based on boundary measurements. In link with this, we refer to recent works in the context of interface problems

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[4,9,16,18,21,23]. The general schematic way, presented in this paper, can be extended to other equations such as, Stokes, Lamé systems.

This paper is organized as follows. In the next section, we give some useful functional spaces, we formulate the underlined problem and we introduce some notations for small perturbations of a surface. In section 3, we derive a formal asymptotic expansion for (tangential) boundary perturbations for both magnetic $\mathbf{H}_\delta \times \nu$ and electric fields $(\nabla \times \mathbf{E}_\delta) \times \nu$ once $\delta \to 0$. In section 4, we present a relationship between boundary measurements and the shape deformation h.

2. Problem formulation and preliminary results

2.1. Functional spaces

In this paper, we denote by bold letters the functional spaces for the vector fields in $\mathbb{R}^d$. Thus, we denote by $\mathcal{D}(\Omega)$ the space of the 3D vector fields with each component belonging to $C_0^\infty(\Omega)$ and by $\mathcal{D}'(\Omega)$ the corresponding dual space. The duality is denoted by $< \cdot, \cdot >_{\mathcal{D}}$. Moreover, $H^s(\Omega)$ denotes the usual Sobolev space on Ω and $\mathbf{H}^s(\Omega)$ denotes $(H^s(\Omega))^d$ and $\mathbf{L}^2(\Omega)$ denotes $(L^2(\Omega))^d$. As usual for Maxwell equations, we recall that if the domain Ω is regular, all the definitions here below make sense and are correct (see for example, [10,11]). Throughout this paper, we denote by ν the outward unit normal to Ω. Let us set

$$
\begin{aligned}
H(\mathrm{curl}, \Omega) &:= \{\mathbf{v} \in \mathbf{L}^2(\Omega); \mathrm{curl}\, \mathbf{v} \in \mathbf{L}^2(\Omega)\}, \quad \|\cdot\|_{H(\mathrm{curl},\Omega)} \text{ the graph norm,} \\
H(\mathrm{div}, \Omega) &:= \{\mathbf{v} \in \mathbf{L}^2(\Omega); \mathrm{div}\, \mathbf{v} \in \mathbf{L}^2(\Omega)\}, \quad \|\cdot\|_{H(\mathrm{div},\Omega)} \text{ the graph norm,} \\
&:= \{\mathbf{v} \times \nu : \mathbf{v} \in H(\mathrm{curl}, \Omega)\} \\
\mathbf{H}^s(\partial\Omega) &:= (H^s(\partial\Omega))^d \quad \text{for } s > 0, \text{ and } \mathbf{H}^0(\partial\Omega) := \mathbf{L}^2(\partial\Omega).
\end{aligned}
$$

Now, using previous definitions we can define the Hilbert space

$$
TH_{\mathrm{div}}^{-1/2}(\partial\Omega) := \{\mathbf{v} \in \mathbf{H}^{-1/2}(\partial\Omega); \mathbf{v} \cdot \nu = 0, \mathrm{div}_{\partial\Omega}\, \mathbf{v} \in \mathbf{H}^{-1/2}(\partial\Omega))\}
$$

endowed with the norm

$$
\|\cdot\|_{TH_{\mathrm{div}}^{-1/2}(\partial\Omega)} = \|\cdot\|_{H^{-1/2}(\partial\Omega)} + \|\mathrm{div}\cdot\|_{H^{-1/2}(\partial\Omega)}.
$$

2.2. Problem formulation

Suppose that an electromagnetic medium occupies a bounded domain $\Omega \subset \mathbb{R}^d$, with a connected C^d-boundary $\partial\Omega$ (for $d = 2, 3$). Let μ_0 and ε_0 denote the permeability and the permittivity of the background medium, and assume that $\mu_0 > 0$ and $\varepsilon_0 > 0$ are positive constants.

Let S, with C^d-boundary, be a conductivity inhomogeneity inside Ω. Let $\mu_1 > 0$ and $\varepsilon_1 > 0$ denote the permeability and the complex permittivity of S. These parameters are also assumed to be positive constants.

In order to prove the main results in this paper, we shall make some regularity assumptions about the region S. We assume that S is C^d, and that there exists a constant $C_0 > 1$ such that

$$
dist(S, \partial\Omega) \geq C_0. \tag{2.1}
$$

Let μ_* and ε_* be the constitutive parameters of the inhomogeneity defined by $\mu_* = \mu_0 + (\mu_1 - \mu_0)\chi_S$, and $\varepsilon_* = \varepsilon_0 + (\varepsilon_1 - \varepsilon_0)\chi_S$ where χ_S denotes the characteristic function for the set S.

We denote by $k = \omega\sqrt{\varepsilon_* \mu_*} > 0$ the wave number, where $\omega > 0$ is a given frequency.