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Spreading and vanishing in a diffusive prey–predator model with variable intrinsic growth rate and free boundary ☆

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ABSTRACT

We study the spreading and vanishing phenomena in a diffusive prey–predator system with variable intrinsic growth rate and free boundary. In this model, the free boundary represents the spreading front and is caused only by the prey, and the variable intrinsic growth rate is allowed to tend to zero and decay "very fast" as $t \to \infty$ or $x \to \infty$. Our main attention is on the effect of variable intrinsic growth rate on the solution and attempt to find some new techniques to deal with the variable intrinsic growth rate. We first study the long time behavior of (u, v) for the vanishing case ($h_\infty < \infty$). Then we find the criteria for spreading and vanishing. At last, the long time behavior of (u, v) for the spreading case ($h_\infty = \infty$) is discussed. Theorems 2.2, 2.3 and 4.1 together establish a spreading–vanishing dichotomy.

1. Introduction

A variety of reaction–diffusion equations are used to describe some phenomena arising in population ecology. A typical one is the diffusive prey–predator model (under the suitable rescaling)

$$\begin{cases} u_t - \Delta u = u(1 - u - av), \\ v_t - d\Delta v = r(t,x)v(b - v + cu), \end{cases} \tag{1.1}$$

where the function $r(t, x)$ is the variable intrinsic growth rate for predator (cf. [23]).

Understanding of spatial and temporal behaviors of interacting species in ecological systems is a central issue in population ecology. One aspect of great interest for a model with multi-species interactions is whether the species can spread successfully. Many mathematicians have made efforts to develop various models and investigated them from a viewpoint of mathematical ecology. When the intrinsic growth rate

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$r(t,x) \equiv r$ is a positive constant, to describe the spreading phenomenon of (1.1), there have been many interesting studies on the existence of positive traveling wave solutions connecting two different equilibria; see, for example [21] and the references cited therein. Also, the study of asymptotic spreading speed plays an important role in population ecology since it can be used to predict the mean spreading rate of species. The concept of asymptotic spreading speed comes from Aronson and Weinberger [1,2] and then Lin and Pan [20,24] extended the result of asymptotic spreading speed to system (1.1) when $r(t,x) \equiv r$ is a positive constant.

In this paper we shall study the spreading and vanishing properties of (1.1) by means of a free boundary problem.

Formulation of the Mathematical Model: We know that, in the real world, the following phenomena happen constantly:

(i) One kind of plant diseases and insect pests (the indigenous species, prey) occurs in a certain zone (initial habitat). In order to do the pest control, the most economical, effective and environment-friendly strategy is to use the biological prevention and cure, and put one kind of natural enemies (the new or invasive species, predator) in this district;

(ii) There is one kind of species (indigenous species, prey) in a certain area (initial habitat), and at some time (initial time) another kind of species (new or invasive species, predator) intrudes into this region.

In view of the diversity of food, the predator, in addition to such prey considered here, has other natural sources of food. It is reasonable to use the diffusive prey–predator model (1.1) describing the interaction between such two species.

To obtain the better and larger habitat, both prey and predator have a tendency to emigrate from the boundary (front) to obtain their new habitat, i.e., they will move outward along the unknown curve (free boundary) as time increases. As a general rule, to avoid being hunted, the prey will have a stronger tendency than the predator. It makes sense to assume that the free boundary is caused only by the prey and the spreading front expands at a speed that is proportional to the prey's population gradient at the front [3,34]. We argue that such prey in this model is the most favored food of the predator as causes of the *features of partial eclipse and picky eaters* for species, and its spreading behavior has such a dominant influence of spreading of the predator that they roughly share the same spreading front. For simplicity, we focus on our problem to the one dimensional case. And for the sake of clarity, it is assumed that the left boundary is fixed ($x = 0$) and the right boundary is free which is given by $x = h(t)$. We shall take the homogeneous Neumann boundary condition at the left boundary $x = 0$ for the prey, and homogeneous Neumann boundary conditions at both the left boundary $x = 0$ and free boundary $x = h(t)$ for the predator.

In consideration of the above reasons, we shall use the following free boundary problem

$$\begin{cases} u_t - u_{xx} = u(1-u-av), & t>0, \quad 0<x<h(t), \\ v_t - dv_{xx} = r(t,x)v(b-v+cu), & t>0, \quad 0<x<h(t), \\ u_x = v_x = 0, & t \ge 0, \quad x=0, \\ u = v_x = 0, \quad h'(t) = -\mu u_x, & t \ge 0, \quad x = h(t), \\ h = h_0, \ u = u_0(x), \ v = v_0(x), & t=0, \ 0 \le x \le h_0 \end{cases} \tag{1.2}$$

to describe the dynamical properties of prey, predator and free boundary. Here, a, b, c, d, h_0 and μ are given positive constants. We assume that the variable intrinsic growth rate $r(t,x)$ satisfies

(H) The function $r(t,x)$ is bounded and positive for $t, x \ge 0$, and $r_x \in C([0,\infty)\times[0,\infty))$, $r \in C^{\frac{\alpha}{2},\alpha}([0,\infty)\times[0,\infty))$ for some $\alpha \in (0,1)$.

The initial functions $u_0(x)$, $v_0(x)$ satisfy

$$u_0, \, v_0 \in C^2([0,h_0]), \ u_0'(0) = v_0'(0) = u_0(h_0) = v_0'(h_0) = 0, \ u_0(x) > 0 \text{ in } [0,h_0) \text{ and } v_0(x) > 0 \text{ on } [0,h_0].$$