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Homogenization results for micro-contact elasticity problems

Anca Capatina[a], Claudia Timofte[b,*]

[a] Institute of Mathematics of the Romanian Academy, P.O. Box 1-764, RO-70700 Bucharest, Romania
[b] University of Bucharest, Faculty of Physics, Bucharest-Magurele, P.O. Box MG-11, Romania

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ABSTRACT

The asymptotic behavior of some elasticity problems, in a perforated domain, is analyzed. We address here the case of an ϵ-periodic perforated structure, with rigid inclusions of the same size as the period. The body occupying this domain is considered to be clamped along a part of its outer boundary and subjected to given tractions on the rest of the exterior boundary. Several nonlinear conditions on the boundary of the rigid inclusions are considered. The approach we follow is based on the periodic unfolding method, which allows us to deal with general materials.

1. Introduction

The behavior of heterogeneous materials, with inhomogeneities at a length scale which is much smaller than the characteristic dimensions of the system, is of huge interest in the theory of composite materials. The homogenization theory was successfully applied for modeling the behavior of such materials, leading to appropriate macroscopic continuum models, obtained by averaging the rapid oscillations of the material properties. Besides, such effective models have the advantage of avoiding extensive numerical computations arising when dealing with the small scale behavior of the system.

This paper deals with the derivation of macroscopic models for some elasticity problems in periodically perforated domains with rigid inclusions of the same size as the period. This periodic structure is occupied by a linearly elastic body which is considered to be clamped along a part of its outer boundary. On the rest of the exterior boundary, surface tractions are given. The body is subjected to the action of given volume forces. Several nonlinear conditions on the boundary of the rigid inclusions are considered. More precisely, we study the case when a nonlinear Robin condition is imposed and, respectively, the case when unilateral contact with given friction is taken into consideration. By using the periodic unfolding method, introduced by Cioranescu, Damlamian & Griso [3] and by Cioranescu, Donato & Zaki [6,7] (see, also, [4]), we obtain the corresponding macroscopic problems.

* Corresponding author.
E-mail addresses: anca.capatina@imar.ro (A. Capatina), claudiatimofte@yahoo.com (C. Timofte).

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Similar problems have been addressed, using various tools and techniques, by many authors. The macroscopic behavior of a composite material with two elastic components is analyzed, in a formal way, by Léné & Leguillon in [17] and [18]. The homogenization of a contact problem of Signorini type in elasticity, by using the two-scale method, is, for the first time, addressed by Mikelić, Shillor & Tapiéro in [22]. In [14], Iosif'yan studies the asymptotic behavior of the solution of a classical problem in elasticity for a perforated body, clamped along its outer boundary and with a Signorini condition imposed on the surface of the cavities. A viscoelastic periodically perforated material with rigid inclusions, for which the contact and the friction are described by linear conditions, is considered by Gilbert, Panchenko & Xie in [12]. For an homogenized model for acoustic vibrations of composite materials with internal friction, the interested reader is referred to Gilbert, Panchenko & Xie [13]. A system of linear elasticity is considered in [14] for a periodically perforated domain in the case in which a nonlinear Robin condition is imposed on the boundary of the inclusions. Recently, Cioranescu, Damlamian & Orlik [5] perform the homogenization, via the periodic unfolding method, of a contact problem for an elastic body with closed and open cracks. Their problem involves the jump of the solution on the oscillating interface. For elasticity problems involving jumps at imperfect interfaces, see also Ene & Pasa [11], Lipton & Vernescu [19,20], and Mei & Vernescu [21].

In this paper, for the Robin problem, we extend, via the periodic unfolding method, some of the results contained in [12] and [14], by considering general nonlinearities in the condition imposed on the boundary of the inclusions. Also, we establish an homogenization result for a Signorini problem with Tresca friction. The difficulties of this problem come from the fact that the unilateral condition generates a convex cone of admissible displacements, and, especially, from the fact that the friction condition involves a nonlinear functional containing the norm of the tangential displacement on the boundary of the rigid inclusions. As shown in Section 4, the macroscopic problem is different from the one addressed in [5]. In particular, for the frictionless contact case, we regain a result obtained, under more restrictive assumptions, in [14]. This frictionless problem was also addressed in [15], by the two-scale convergence method, for more general geometric structures of the inclusions on which the Signorini conditions act.

The structure of the paper is as follows: in Section 2, we formulate our microscopic problems, namely a nonlinear Robin problem and a Signorini–Tresca one. Section 3 is devoted to the homogenization of the Robin problem. In Section 4, we obtain the macroscopic behavior of the solution of the Signorini–Tresca problem.

2. The microscopic problems

Let us consider a linearly elastic body occupying a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$ (the relevant physical cases are $n = 2$ or $n = 3$), with a Lipschitz boundary $\Gamma = \overline{\Gamma}_1 \cup \overline{\Gamma}_2$, where Γ_1, Γ_2 are open and disjoint parts of Γ, with $meas(\Gamma_1) > 0$.

The body is subjected to the action of a volume force of density $\boldsymbol{f}$ given in Ω and a surface traction of density $\boldsymbol{t}$ applied on Γ_2. The body is clamped on Γ_1 and, so, the displacement vector $\boldsymbol{u}$ vanishes here.

Let $Y = (0,1)^n$ be the representative cell and T, the rigid part, be an open subset of Y, with a Lipschitz boundary ∂T and such that $\bar{T} \subset Y$. Let $Y^* = Y \backslash \bar{T}$ be the elastic part. We assume that the set of all translated images of $\bar{T}$ of the form $\epsilon(\mathbf{l} + \bar{T})$, with $\mathbf{l} \in \mathbb{Z}^n$, does not intersect the boundary $\partial\Omega$. We denote by T_ϵ the set of the inclusions contained in Ω, i.e.

$$T_\epsilon = \bigcup_{\mathbf{l} \in \mathcal{K}_\epsilon} \epsilon(\mathbf{l} + T),$$

where $\mathcal{K}_\epsilon = \{\mathbf{l} \in \mathbb{Z}^n / \epsilon(\mathbf{l} + \bar{T}) \subset \Omega\}$.

We define the periodic perforated domain by

$$\Omega_\epsilon = \Omega \backslash \bar{T}_\epsilon.$$