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Energy evolution of multi-symplectic methods for Maxwell equations with perfectly matched layer boundary

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ABSTRACT

In this paper, we consider the energy evolution of multi-symplectic methods for three-dimensional Maxwell equations with perfectly matched layer boundary, and present the energy evolution laws of Maxwell equations under the discretization of multi-symplectic Yee method and general multi-symplectic Runge–Kutta methods.

1. Introduction

Maxwell equations are the most foundational equations in electromagnetism and are widely applied to many application fields, such as aeronautics, electronics, and biology [12,13], etc. They are mathematical expressions of the natural laws correlative fields, such as Ampère's law and Faraday's law [14]. On the other hand, in lossless medium, the electromagnetic energy of the wave is constant at different times [4]. As we all know, to preserve the energy is greatly important in constructing numerical schemes for different physical problems. However, numerical methods, with some boundary conditions, cannot preserve the energy exactly in general cases. Therefore, it is important and necessary to investigate the energy evolution of Maxwell equations under numerical discretization with some boundary conditions. The purpose of this paper is to study the energy evolution of multi-symplectic methods for three-dimensional (3D) Maxwell equations with perfectly matched layer (PML) boundary.

It has been recognized that symplectic structure-preserving numerical methods have significant superiority over non-symplectic methods in numerical solving Hamiltonian ordinary differential equations (ODEs) and Hamiltonian partial differential equations (PDEs) [3]. At the end of last century, symplectic integrators

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have been generalized to multi-symplectic ones [4–9,11]. And multi-symplectic integrators have been applied to Maxwell equations. For example, Ref. [11] discussed the self-adjointness of the Maxwell equations with variable coefficients ϵ and μ, and showed that the equations have the multi-symplectic structure. Ref. [4] firstly proposed an unconditionally stable, energy-conserved, and computationally efficient scheme for two-dimensional (2D) Maxwell equations with an isotropic and lossless medium. The further analysis in the case of 3D was studied in [5]. Meanwhile, Ref. [9] proposed a kind of splitting multi-symplectic integrators method for Maxwell equations in three dimensions, which was proved to be unconditionally stable, non-dissipative, and of first order accuracy in time and second order accuracy in space.

It is well known that the PML boundary conditions are widely applied to the numerical simulation Maxwell equations. In 1993, Berenger [1,2] firstly proposed the PML technique, which is based on modifying the PDEs away from all physical boundaries such that absorbing outgoing waves from the computation domain. It is a simple and straightforward technique, easily implemented for both two and three space dimensions using either Cartesian or cylindrical coordinates. However, to the best of our knowledge, the investigation of multi-symplectic methods for Maxwell equations with PML boundary does not exist. In this paper, inspired by this problem, we investigate the energy evolution of general multi-symplectic methods for Maxwell equations with Berenger's PML boundary.

The rest of this paper is organized as follows. In Section 2, we begin with some preliminary results about 3D Maxwell equations and Berenger's PML systems. An equivalent formulation to Berenger's PML systems is introduced in Section 3. In Section 4, we present the energy evolution laws of multi-symplectic Yee method and general multi-symplectic Runge–Kutta methods for 3D Maxwell equations with PML boundary. Finally, concluding remarks are presented in Section 5.

2. Preliminary results

2.1. 3D Maxwell equations

For a linear homogeneous medium within linear isotropic material with the permittivity ε and the permeability μ, the scattering of electromagnetic waves without the charges or the currents can be described by the 3D Maxwell equations in curl formulation

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon}\nabla \times \mathbf{H}, \tag{2.1}$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu}\nabla \times \mathbf{E}, \tag{2.2}$$

where $\mathbf{E} = (E_x, E_y, E_z)^T$ and $\mathbf{H} = (H_x, H_y, H_z)^T$ represent the electric field and the magnetic field, respectively. The domain $\Omega \times [0, T] = [0, a] \times [0, b] \times [0, c] \times [0, T]$ under consideration is occupied by this medium and surrounded by perfect conductors.

Equations (2.1) and (2.2) can be written as the componentwise formula

$$\frac{\partial}{\partial t}\begin{bmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \frac{1}{\varepsilon}(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}) \\ \frac{1}{\varepsilon}(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}) \\ \frac{1}{\varepsilon}(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \\ \frac{1}{\mu}(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}) \\ \frac{1}{\mu}(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}) \\ \frac{1}{\mu}(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}) \end{bmatrix}. \tag{2.3}$$