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On the null controllability of heat equation with memory

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ABSTRACT

The aim of this article is to analyze the null controllability properties of the heat equations with memory. By a new Carleman inequality with weighted functions that do not vanish at $t = 0$, we prove the controllability for the system with a located internal controller under a condition on the kernel.

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1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^N$ with the smooth boundary Γ and $T > 0$. Put $Q = \Omega \times (0, T)$ and $\Sigma = \Gamma \times (0, T)$. Let χ_ω stand for a characteristic function of a nonempty open subset ω of Ω. Consider the following controlled heat equation with memory

$$\begin{cases} y_t - \Delta y = \int_0^t a(x,t,s)y(x,s)ds + \chi_\omega u, & \text{in } Q, \\ y(x,t) = 0, & \text{on } \Sigma, \\ y(x,0) = y_0(x), & \text{in } \Omega, \end{cases} \tag{1.1}$$

where y and u are the state variable and the control variable respectively, a is a given L^∞ function defined on $\Omega \times (0, T) \times (0, T)$.

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These types of equations arise in many fields such as heat conduction in materials with memory, population dynamics, nuclear reactors, etc. (see [16,23,4,24] for instance).

In this paper, our main interest concerns the controllability properties of (1.1). The aim is to study whether, for any given initial y_0, there exist controls such that the associated solutions satisfy appropriate requirements at given time $t = T$.

In the last two decades, there have appeared many works addressing the controllability problems of classical parabolic equations subject to appropriate initial and boundary conditions

$$y_t - \Delta y + ay = \chi_\omega u,$$

where ω be a given nonempty open subset of Ω such that $\overline{\omega} \subseteq \Omega$ (we refer to [7,8,6,5,12,2,1] and the references therein). However, to our knowledge, very few results are known regarding the controllability of the parabolic equations with memory, see [3,14,10,15,22,18,13]. Barbu and Iannelli [3] obtained the approximate controllability of a multidimensional system with memory involving the second order spatial derivative

$$y_t - \gamma \Delta y - \int_0^t a(t-s)\Delta y(x,s)ds = \chi_\omega u. \tag{1.2}$$

And they also studied the exact controllability of the one-dimensional linear viscoelasticity equation

$$y_t - \int_0^t a(t-s)y_{xx}(x,s)ds = \chi_\omega u. \tag{1.3}$$

In both cases, they assumed that $a \in C^\infty(0, +\infty)$ is a locally integrable completely monotone kernel. Under this condition, the system has certain representations by the Laplace transform. Then, Halanay and Pandolfi [14] extended the approximate controllability to every C^1 kernel. Fu et al. [10] considered the heat equation with hyperbolic memory kernel

$$y_t - \sum_{i,j}\left(a^{i,j}\int_0^t b(x,t-s)y_{x_i}(x,s)ds\right)_{x_j} = \chi_\omega u. \tag{1.4}$$

It was shown in [10] that the heat equation with such kind of memory admits a finite speed of propagation for heat pulses. Hence, the corresponding controllability of (1.4) is similar to that of hyperbolic equations, and therefore, significantly different from the controllability of the usual parabolic equations. In [15], for a one-dimensional heat equation with memory, the authors proved the lack of “controllability to the rest” for some initial conditions with both distributed and boundary controls. In [22,18], the authors obtain the null controllability of the system

$$y_t - \Delta y + \int_0^t k(t,s)f(y(x,s))ds = \chi_\omega u,$$

where $f(y)$ is a globally Lipschitz continuous function. The proof relies on a Carleman inequality which requires that

$$\operatorname{supp} k(\cdot, s) \subset (t_0, t_1),\ 0 < t_0 < t < t_1 < T,\ \forall s \in (0, T).$$

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