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Heat–wave interaction in 2–3 dimensions: Optimal rational decay rate

George Avalos [a], Irena Lasiecka [b,c], Roberto Triggiani [b,*]

[a] *Department of Mathematics, University of Nebraska–Lincoln, USA*
[b] *Department of Mathematical Sciences, University of Memphis, USA*
[c] *IBS, Polish Academy of Sciences, Warsaw, Poland*

A R T I C L E I N F O

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This paper is dedicated to the memory of David L. Russell, a pioneer of control theory of PDEs and long-time colleague, mentor and friend

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A B S T R A C T

In this paper, we consider a simplified version of a fluid–structure PDE model which has been of longstanding interest within the mathematical and biological sciences. In it, a n-dimensional heat equation replaces the original Stokes system, so as to ultimately have a vector-valued heat equation and vector-valued wave equation compose the coupled PDE system under study. The coupling between the two disparate PDE components occurs across a boundary interface. As such, the entire PDE dynamics manifests features of both hyperbolicity and parabolicity. For this heat–structure system, our main result of uniform stability is as follows: Given smooth initial data – i.e., data in the domain of the underlying semigroup generator of the coupled PDE system – the corresponding solutions decay at the rate of $o(t^{-1})$. This establishes the long-conjectured optimal rate. The problem of obtaining sharp rational decay rates for the heat–wave PDE, under present consideration, has been a much considered problem, with the *modus operandi* of earlier efforts taking place within the time domain. By contrast, we adopt here a frequency domain approach which is based upon a recent resolvent criterion, and which was initiated in our prior effort, wherein we obtained the rate of decay $o(\frac{1}{\sqrt{t}})$. The present optimal improvement $o(t^{-1})$ is achieved by employing an additional tool in our analysis – a microlocal analysis argument – to estimate a critical term involving two problematic boundary traces.

1. Introduction and statement of main result

Introduction. We proceed to describe the canonical heat–structure PDE model of the present paper. This is the first step toward the more realistic fluid–structure PDE model which has the more challenging dynamic Stokes equation in place of the n-dimensional heat equation [42, p. 121], [19]. It will be treated in a subsequent publication [6], which will take advantage of the present treatment, in order to tackle the

* Corresponding author.
E-mail address: R.Triggiani@memphis.edu (R. Triggiani).

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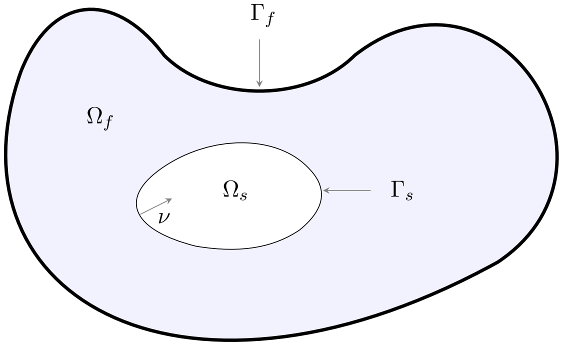

Fig. 1. The fluid–structure interaction.

additional challenge due to the presence of the pressure. Throughout, $\Omega_f \subseteq \mathbb{R}^n$, $n = 2$ or 3, will denote the bounded domain on which the heat component of the coupled PDE system evolves. Its boundary $\partial\Omega_f = \Gamma_s \cup \Gamma_f$, $\Gamma_s \cap \Gamma_f = \emptyset$, with each boundary piece being sufficiently smooth. Moreover, the geometry Ω_s, immersed within Ω_f, will be the domain on which the structural component evolves with time. As configured then, the coupling between the two distinct fluid and elastic dynamics occurs across boundary interface $\Gamma_s = \partial\Omega_s$; see Fig. 1. In addition, the unit normal vector $\nu(x)$ will be directed away from Ω_f, and so *toward* Ω_s. (This specification of the direction of ν will influence the computations to be done below.)

On this geometry in Fig. 1, we thus consider the following heat–structure PDE model in solution variables $u = [u_1(t,x), u_2(t,x), \ldots, u_n(t,x)]$ (the heat component, replacing the fluid velocity field), and $w = [w_1(t,x), w_2(t,x), \ldots, w_n(t,x)]$ (the structural displacement field):

$$\begin{aligned}
&(\text{PDE}) \begin{cases} u_t - \Delta u = 0 & \text{in } (0,T)\times\Omega_f, \quad (a) \\ w_{tt} - \Delta w + w = 0 & \text{in } (0,T)\times\Omega_s; \quad (b) \end{cases} \\
&(\text{BC}) \begin{cases} u|_{\Gamma_f} = 0 & \text{on } (0,T)\times\Gamma_f, \quad (c) \\ u = w_t & \text{on } (0,T)\times\Gamma_s, \quad (d) \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial w}{\partial \nu} & \text{on } (0,T)\times\Gamma_s; \quad (e) \end{cases} \\
&(\text{IC})\ [w(0,\cdot), w_t(0,\cdot), u(0,\cdot)] = [w_0^*, w_1^*, u_0^*] \in \mathbf{H}, \quad (f)
\end{aligned} \tag{1.1}$$

where space of well-posedness is taken to be the finite energy space

$$\mathbf{H} \equiv \mathbf{H}^1(\Omega_s) \times \mathbf{L}^2(\Omega_s) \times \mathbf{L}^2(\Omega_f), \tag{1.2}$$

for the variable $[w_1, w_2, u]$. (We are using the common notation $\mathbf{H}^s \equiv [H^s]^n$.) Of course, $\mathbf{H}$ is a Hilbert space with the following norm inducing inner product, where $(f,g)_\Omega \equiv \int_\Omega f\overline{g}d\Omega$:

$$\left(\begin{bmatrix} v_1 \\ v_2 \\ f \end{bmatrix}, \begin{bmatrix} \tilde{v}_1 \\ \tilde{v}_2 \\ \tilde{f} \end{bmatrix} \right)_{\mathbf{H}} = (\nabla v_1, \nabla \tilde{v}_1)_{\Omega_s} + (v_1, \tilde{v}_1)_{\Omega_s} + (v_2, \tilde{v}_2)_{\Omega_s} + \left(f, \tilde{f}\right)_{\Omega_f}. \tag{1.3}$$

Semigroup well-posedness. As was done in [5,7–13,4] for a fluid–structure system in which Stokes flow is used to describe the fluid component of the dynamics, one can provide a non-trivial semigroup formulation so as to describe the corresponding time-evolving PDE model (1.1)(a)–(f) (as a very special case of the above references, as no pressure occurs now). To this end, one define a modeling generator $\mathcal{A} : \mathbf{H} \to \mathbf{H}$ as follows: