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A 3D boundary optimal control for the bidomain-bath system modeling the thoracic shock therapy for cardiac defibrillation

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ABSTRACT

This work is dedicated to study the cardiac defibrillation problem by using an optimal thoracic electroshock treatment. The problem is formulated as an optimal control problem in a 3D domain surrounded by the bath and including the heart. The control corresponds to the thoracic electroshock and the model describing the electrical activity in the heart is the bidomain model. The bidomain model is coupled with the quasi-static Maxwell's equation to consider the effect of an external bathing medium. The existence and uniqueness of a weak solution for the direct problem is assessed as well as the existence of a weak solution for the adjoint problem. The numerical discretization is realized using a finite element method for the spatial discretization and linearly implicit Runge–Kutta methods for the temporal discretization of the partial differential equations. The numerical results are demonstrated for the termination of re-entry waves.

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1. Introduction

One of the leading causes of death all over the world are cardiovascular diseases. Knowledge and understanding of the electrical heart activity are important issues in order to establish new diagnostic techniques. In this work we would like to use control techniques by mean of high external stimulations over the body thorax in order to steer the heart electrical activity to a given level. This kind of treatment is used in cardiology to reset some arrhythmia disease (for more details see e.g. [32] and [33]). In literature, numerous investigations focused on electrophysiologically important issues such as the formation of reentrant arrhythmias such as spiral waves [17] and their degeneration into fibrillation [28], or the termination of turbulent

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electrical activity in the heart by applying strong electric fields (defibrillation) [4], the only known therapy to terminate otherwise lethal ventricular fibrillation.

In order to model the electrical heart activity, we distinguish the geometry of the bath and the electrical activation in the myocardium which is based on the bidomain model. This model was introduced by Tung [29] and it is widely considered to be among the most complete descriptions of bioelectric activity at the tissue and organ level [30,8]. The electrical heart activity and the volume conductor for the bath model is given by the following system:

$$\begin{cases} \beta c_m \partial_t u - \nabla \cdot (\mathbf{M}_{\mathrm{i}}(x)\nabla u_i) + \beta I_{\mathrm{ion}}(u,w) = I_i, & (t,x) \in \Omega_{T,H} := (0,T)\times\Omega_H, \\ \beta c_m \partial_t u + \nabla \cdot (\mathbf{M}_{\mathrm{e}}(x)\nabla u_e) + \beta I_{\mathrm{ion}}(u,w) = I_e, & (t,x) \in \Omega_{T,H}, \\ -\nabla \cdot (\mathbf{M}_{\mathrm{s}}(x)\nabla u_s) = 0, & (t,x) \in \Omega_{T,B} := (0,T)\times\Omega_B, \\ \partial_t w - H(u,w) = 0, & (t,x) \in \Omega_{T,H}, \\ (\mathbf{M}_{\mathrm{i}}(x)\nabla u_i)\cdot\eta = 0, & (t,x) \in \Sigma_{T,H} := (0,T)\times\Sigma_H, \\ (\mathbf{M}_{\mathrm{e}}(x)\nabla u_e)\cdot\eta = (\mathbf{M}_{\mathrm{s}}(x)\nabla u_s)\cdot\eta, & (t,x) \in \Sigma_{T,H}, \\ u_e = u_s, & (t,x) \in \Sigma_{T,H}, \\ (\mathbf{M}_{\mathrm{s}}(x)\nabla u_s)\cdot\eta_s = 0, & (t,x) \in \Sigma_{T,B}\backslash\Sigma_{T,H}, \\ u(0,x) = u_0(x), & x \in \Omega_H, \\ w(0,x) = w_0(x), & x \in \Omega_H. \end{cases} \tag{1.1}$$

The heart's spatial domain is represented by $\Omega_H \subset \mathbb{R}^3$ which is a bounded open subset, and by Σ_H we denote its piecewise smooth boundary. A distinction is made between the intracellular and extracellular tissues which are separated by the cardiac cellular membrane. The thorax is modeled by a volume conduction Ω_B with a piecewise smooth boundary $\Sigma_B := \Sigma_H \cup \Sigma$, where Σ is the thorax surface. The whole domain is denoted by the $\Omega = \bar{\Omega}_H \cup \Omega_B$, we refer to Fig. 1 for the pictorial representation of such computational domain and other sub-domains. For all $(x,t) \in \Omega_{T,H} := \Omega_H \times (0,T)$, $u_i = u_i(x,t)$, $u_e = u_e(x,t)$ stand for the intracellular and extracellular potentials respectively, and for all $(x,t) \in \Omega_{T,B} := \Omega_B \times (0,T)$, $u_s(x,t)$ stands for the bathing medium electric potential. The transmembrane potential is the difference $u = u(x,t) := u_i - u_e$. $\mathbf{M}_{\mathrm{i}}(x)$ and $\mathbf{M}_{\mathrm{j}}(x)$ are tensors which represent respectively the intracellular and extracellular conductivity of the tissue respectively. The diagonal matrix $\mathbf{M}_{\mathrm{s}}$ represents the conductivity tensor of the bathing medium.

The constant $c_m > 0$ is the surface capacitance of the membrane and β is the surface-to-volume ratio.

We denote by I_i and I_e the internal and the external current stimulus respectively. Moreover, $H(u,w)$ and $I_{\mathrm{ion}}(u,w)$ are functions which correspond to the widely known FitzHugh–Nagumo model for the membrane and ionic currents (see e.g. [14,22]). For detailed exposition of the such several ionic models we refer to [15] and [27]. Recalling the definition of $H(u,w)$ and $I_{\mathrm{ion}}(u,w)$, we know from [14] and [22] that the membrane kinetics can be simply reformulated by:

$$H(u,w) = au - bw, \tag{1.2}$$

$$I_{\mathrm{ion}}(u,w) = -\lambda(w - u(1-u)(u-\theta)), \tag{1.3}$$

where a, b, λ, θ are given parameters. Moreover, we impose the following zero mean condition for the extracellular potential in order to obtain uniqueness of the elliptic systems:

$$\int_{\Omega_H} u_e(t,x)\mathrm{d}x = 0, \quad \text{for all } t \in (0,T). \tag{1.4}$$

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