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Time-consistent investment-reinsurance strategy for mean-variance insurers with a defaultable security

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ABSTRACT

This paper considers an optimal investment and reinsurance problem involving a defaultable security for an insurer under the mean-variance criterion in a jump-diffusion risk model. The insurer can purchase proportional reinsurance or acquire new insurance business and invest in a financial market consisting of a risk-free asset, a stock and a defaultable bond. In particular, the correlation between the insurance risk model and the financial market is also considered. From a game theoretic perspective, the extended Hamilton–Jacobi–Bellman systems of equations are established for the post-default case and the pre-default case, respectively. In both cases, closed-form expressions for the optimal time-consistent investment-reinsurance strategies and the corresponding value functions are derived. Moreover, some properties of optimal strategies, value functions and efficient frontiers are discussed either analytically or numerically.

1. Introduction

Nowadays, investment and reinsurance are playing increasingly important roles in insurance business. There has been emerging interest on optimal reinsurance and investment problems for insurers. Most existing works adopted the utility maximization or the ruin probability minimization as objective functions. For example, Browne [13] considered a diffusion risk model and obtained investment strategies of maximizing the exponential utility or minimizing the probability of ruin. Adopting similar objective functions, subsequent works extended [13] extensively in different directions. For example, Yang and Zhang [34] studied an optimal investment problem for an insurer with a jump-diffusion risk process. Bai and Guo [2] investigated an optimal proportional reinsurance and investment problem with multiple risky assets and short-selling constraint. Liang et al. [26] and Lin and Li [28] investigated an optimal proportional reinsurance and investment problem of the exponential utility maximization for the jump-diffusion risk process under the constant elasticity of

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variance (CEV) model. Gu et al. [19] studied an optimal excess-of-loss reinsurance and investment problem for the exponential utility maximization under the CEV model.

Under the criterion of minimizing the probability of ruin, Hipp and Plum [20] studied an optimal investment problem under a compound Poisson risk model. Liu and Yang [29] extended the model of [20] to incorporate a non-zero interest rate and derived numerical solutions. Promislow and Young [30] obtained optimal investment and reinsurance strategies to minimize the ruin probability for a diffusion risk model. Belkina et al. [5] investigated the optimal investment problem in the Cramér–Lundberg model under the constraint that the insurance company can only invest in the risky asset at a limited leveraging level. Azcue and Muler [1] considered that the insurer can allocate the reserve in non-cash assets and cash. Under the assumption of transaction costs and the criterion of minimizing the ruin probability, the authors characterized the value function as the unique viscosity solution of the associated Hamilton–Jacobi–Bellman (HJB) equation.

Recently, optimal reinsurance and investment problems under the mean-variance criterion have drawn much attention. Bäuerle [4] investigated an optimal proportional reinsurance/new business problem under the mean-variance criterion for a compound Poisson risk model. Wang et al. [33] used the martingale approach to derive closed-form strategies for an insurer in the mean-variance paradigm. Delong and Gerrard [17] modeled the claim process by a compound Cox process with a drifted Brownian motion intensity and studied the mean-variance portfolio problem involving a running cost. Bai and Zhang [3] derived viscosity solutions for an investment and reinsurance problem of mean-variance optimization. Chiu and Wong [16] considered the continuous-time mean-variance asset-liability management problem for an insurer investing in an incomplete financial market with cointegrated assets. In Bi and Guo [6], a reinsurance and investment problem under no-shorting constraint in a jump-diffusion financial market is studied for a mean-variance insurer. Chen and Yam [15] investigated an optimal investment-reinsurance problem for mean-variance insurers in the market with regime-switching. Shen and Zeng [31] solved an optimal investment-reinsurance mean-variance problem under stochastic volatility models by means of backward stochastic differential equations.

It is known that the dynamic mean-variance problem is time-inconsistent in the sense that Bellman optimality principle does not hold. As illustrated in [9,10], there are two approaches to handle the time inconsistency issue. One approach is to study the corresponding precommitment problem, while the other is to formulate the problem in a game theoretic framework. In all of the aforementioned literature on mean-variance problems for insurers, time-inconsistent strategies are derived in the precommitment formulation. Since time consistency of strategies is a basic requirement for rational decision making, it seems more appealing to find time-consistent strategies for mean-variance problems. The game theoretic approach is tailor-made to find time-consistent strategies (i.e., sitting at time t, the optimal strategy derived at time t should agree with the optimal strategy derived at time $t + \Delta t$). Recently, Zeng and Li [35] derived the time-consistent investment and reinsurance strategies for mean-variance insurers. Wang and Forsyth [32] provided a numerical scheme for determining the time-consistent strategy of mean-variance optimization. Zeng et al. [36] obtained the time-consistent investment and reinsurance strategies for a mean-variance insurer with a general jump-diffusion risk model, where the logarithm of risky asset's price follows a jump-diffusion process.

Although optimal investment-reinsurance problems have been extensively studied, most previous works assumed that the financial market consists of one risk-free asset and one stock/multiple stocks. The credit or default risk is rarely considered in the modeling framework. But in recent years, institutional investors, such as insurers and pension funds, are actively participating in trading high yield bonds with default risk, say, corporate bonds. According to the National Association of Insurance Commissioners's Capital Markets Bureau at year-end 2012,[1] corporate bonds constituted the largest component of investment portfolios across

[1] Please refer to http://www.naic.org/capital_markets_archive/130924.htm for capital market special report: update on insurance industry investment portfolio asset mixes.