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Strain gradient theory of chiral Cosserat thermoelasticity without energy dissipation

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A B S T R A C T

In this paper, we use the Green–Naghdi theory of thermomechanics of continua to derive a linear strain gradient theory of Cosserat thermoelastic bodies. The theory is capable of predicting a finite speed of heat propagation and leads to a symmetric conductivity tensor. The constitutive equations for isotropic chiral thermoelastic materials are presented. In this case, in contrast with the classical Cosserat thermoelasticity, a thermal field produces a microrotation of the particles. The thermal field is influenced by the displacement and microrotation fields even in the equilibrium theory. Existence and uniqueness results are established. The theory is used to study the effects of a concentrated heat source in an unbounded homogeneous and isotropic chiral solid.

1. Introduction

In the theory of continua with inner structure the material particles are considered geometrical points that possess properties similar to rigid particles (Cosserat continua) and deformable particles, capable of undergoing only affine deformations. In the case of Cosserat continua the degree of freedom for each material point is six: three translations and three microrotations. The domain of applicability of the theory of continua with inner structure has been investigated by Fischer-Hjalmars [12], Kunin [28], Eringen [11], Dyszlewicz [10] and Chen et al. [7]. The classical theory of Cosserat elastic solids is characterized by constitutive functions which depend on the deformation gradient, microrotation vector and gradient of microrotation. Rymarz [47] and Brulin and Hjalmars [3] have developed a theory of Cosserat elastic solids where the second-order displacement gradient is added to the classical set of independent constitutive variables. This theory was named the grade consistent micropolar elasticity. The theory has been studied and extended in various papers (see, e.g., Scalia [48]; Ieşan [25]; Zhang and Sharma [52] and references therein). In the absence of microrotation field, the theory reduces to the strain gradient theory of elasticity established by Toupin

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[50,51] and Mindlin [37]. The coupling between the strain gradient theory and the Cosserat theory has been used in the plasticity theory. Thus, Chen and Wang [8] investigated the deformation of thin metallic wire torsion and ultra-thin metallic beam bend. The analytical results agree well with the experimental results.

The purpose of the present paper is to present a strain gradient theory of Cosserat thermoelasticity without energy dissipation and to investigate the chiral effects. Green and Naghdi [18,19] developed a thermomechanical theory of deformable continua that relies on an entropy balance law rather than an entropy inequality. A theory of thermoelastic bodies based on the new entropy balance law has been derived by Green and Naghdi [20]. The linearized form of this theory does not sustain energy dissipation and permits the transmission of heat as thermal waves at finite speed. Moreover, the heat flux vector is determined by the same potential function that determines the stress. The Green–Naghdi theory has been studied in various papers (see, e.g., Chandrasekharaiah [6]; Hetnarski and Ignazack [23]; Quintanilla and Straughan [45,46]; Quintanilla [44]; Puri and Jordan [43]; Ieşan and Quintanilla [27]; Bargmann [2] and references therein). The gradient theories of thermomechanics have been studied in various papers (see, e.g., Ahmadi and Firoozbakhsh [1]; Ieşan [24,25]; Ieşan and Quintanilla [26]; Ciarletta and Ieşan, [9]; Martinez and Quintanilla [36]; Forest et al. [15]; Forest and Amestoy [14]; Forest and Aifantis [13]).

In recent years the study of chiral materials has been received a widespread attention. The mechanical behavior of chiral materials is of interest for the investigation of carbon nanotubes [4,22,5], auxetic materials [30,31,42,49] and bones [34,41]. It is known that the deformation of chiral elastic materials cannot be described within classical elasticity. Various authors have studied the behavior of chiral elastic materials by using the theory of Cosserat elasticity (see, e.g., Lakes [32,33]; Park and Lakes [41], and references therein). The strain gradient theory of elasticity is also an adequate tool to describe the deformation of chiral elastic solids (Papanicolopulos [40] and references therein). In the present paper we consider the chiral effects which appear in the Cosserat theory and in the strain gradient theory of elasticity.

In the first part of the paper we derive the basic equations of the strain gradient theory of Cosserat thermoelasticity without energy dissipation and the constitutive equations of isotropic chiral materials. The field equations are expressed in terms of the displacement, microrotation and thermal fields. It is shown that, in contrast with the classical Cosserat thermoelasticity, a thermal field produces a microrotation of the particles. The thermal field is influenced by the displacement and microrotation fields even in the equilibrium theory. We establish existence and uniqueness results in the dynamic theory of thermoelasticity. The existence result is obtained by means of the semigroup theory. Then we consider the equilibrium theory and study the effects of a concentrated heat source in an unbounded chiral isotropic material. We investigate the influence of chiral coefficients on the displacements, microrotations and thermal field.

2. Basic equations

We consider a body that at time t_0 occupies the properly regular region B of Euclidean three-dimensional space and is bounded by the surface ∂B. The motion of the body is referred to a fixed system of rectangular Cartesian axes Ox_i $(i = 1; 2; 3)$. We denote by n_k the outward unit normal of ∂B. We shall employ the usual summation and differentiation conventions: Greek subscripts are understood to range over the integers $(1, 2)$ whereas Latin subscripts, unless otherwise specified, are understood to range over the integers $(1, 2, 3)$, summation over repeated subscripts is implied, and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. We use a superposed dot to denote partial differentiation with respect to the time. Letters in boldface stand for tensors of an order $p \geq 1$, and if $\mathbf{w}$ has order p, we write $w_{ij\ldots k}$ (p subscripts) for the rectangular Cartesian components of $\mathbf{w}$.

In what follows we study elastic media each material point of which has six degrees of freedom. We denote by u_j the displacement vector and by φ_j the microrotation vector.

Let $\mathcal{P}$ be an arbitrary material volume in the continuum, bounded by a surface $\partial\mathcal{P}$ at time t. We suppose that P is the corresponding region in the reference configuration, bounded by a surface ∂P.