



Oscillations, fluctuation intensity and optimal harvesting of a bio-economic model in a complex habitat



Xue Zhang^{a,b,*}, Shuni Song^a, Jianhong Wu^b

^a College of Science, Northeastern University, Shenyang, Liaoning, 110819, China

^b Center for Disease Modelling, York Institute for Health Research, York University, 4700 Keele Street, Toronto, ON, M3J 1P3, Canada

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ABSTRACT

We investigate the effects of habitat complexity and multi-time delays on dynamics of a bio-economic predator–prey model. The differential–algebraic system theory is applied to transform the bio-economic model into a normal form, so that the local stability and existence of periodic solutions can be examined by varying the delays and the habitat complexity parameter. The direction of Hopf bifurcation and the stability of bifurcated periodic solutions are investigated. We also discuss the effect of fluctuating environment on dynamical behavior of a corresponding stochastic delayed-differential–algebraic system and derive expressions for intensities of population fluctuations. The model is also used to study the optimal harvesting strategy in order to maximize economic profit while sustaining the ecosystem. Numerical simulations are designed to illustrate the effectiveness of theoretical analysis.

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1. Introduction

A predator–prey system, intensively studied in the literature (see [22,23,14,20]), is generally given by

$$\begin{cases} \dot{x}(t) = xg(x) - yf(x, y), \\ \dot{y}(t) = \beta yf(x, y) - dy, \end{cases} \quad (1)$$

where x and y denote the number of preys and predators, respectively. In the model, $g(x)$ is the per capita growth rate of the prey in the absence of predation. The trophic function $f(x, y)$ denotes the amount of prey caught by a predator per unit of time, β is the rate of conversion of nutrients from the prey into the reproduction of the predator and d is the mortality rate of the predator in the absence of prey.

* Corresponding author.

E-mail addresses: zhangxue@mail.neu.edu.cn (X. Zhang), songsn@126.com (S. Song), wujh@mathstat.yorku.ca (J. Wu).

There are different types of functional responses such as the prey-dependent type [25] (including the Holling I–III) and the predator-dependent type [12] (including the Beddington–DeAngelis function and ratio-dependent response). It is also noted that (see [5,17,19]) habitat complexity can reduce the probability of capturing a prey by decreasing encounter rates between predator and prey. This led to incorporating the influence of habitat complexity into the Holling II [11] type functional response as follows:

$$f(x) = \frac{\alpha(1 - \delta)x}{1 + \alpha(1 - \delta)\gamma x}, \tag{2}$$

where α and γ denote the attack coefficient and handling time for predation, respectively. The constant δ ($0 < \delta < 1$) is a nondimensional parameter that reflects the strength of habitat complexity. When the habitat complexity is ignored, i.e., $\delta = 0$, the function (2) reduces to the classic Holling Type II functional response. Our work here is based on the aforementioned functional response.

It is well known that extensive and unregulated harvesting can cause species extinction, leading to the destruction of a natural predator–prey ecosystem. Regulated harvesting thus becomes a necessity to maintain an interactive biological system. However, such a regulation is always influenced by the cost-benefit of the harvesting activities. There has already an increasing body of literature for the modelling and analysis of bio-economic systems, often described by differential–algebraic equations (see, for example, [26,3,27,4,18] and references therein). In particular, in [26], a stage-structure differential–algebraic predator–prey system subject to harvesting is proposed to investigate the effects of harvesting on population dynamics. A singularly induced bifurcation leading to impulses and stability switch occurs at some critical point of economic interest, yielding rapid expansion of the predator. Zhang et al. [27] studied a ratio-dependent prey–predator singular model and analyzed the direction and stability of periodic solutions. However, this work ignored the fact that biological processes normally do not take place instantaneously due to the interaction with environment and other species, such as gestation, maturity and hunting. Chakraborty et al. [3] introduced a single discrete gestation delay in a differential–algebraic bio-economic system and established Hopf bifurcations in the neighborhood of coexisting equilibrium point. Liao et al. [15] investigated Hopf bifurcations of a three-species predator–prey system with two delays. In their study, it is possible to rescale the time to regard the sum of these two delays as a natural bifurcation parameter. This idea was also utilized by Song et al. [21] and Ma [16], while other relevant studies such as [24,8] simplified their analysis by requiring the two delays be identical. However, since the delays describing different ecological interaction are always different, it is important to discuss the impact of each delay independently on the dynamics, respectively.

In this paper, we study the following differential–algebraic bio-economic model with two time delays and habitat complexity:

$$\begin{cases} \frac{dx}{dt} = rx \left(1 - \frac{x(t - \tau_1)}{K} \right) - \frac{\alpha(1 - \delta)xy}{1 + \alpha\gamma(1 - \delta)x}, \\ \frac{dy}{dt} = \frac{\beta\alpha(1 - \delta)x(t - \tau_2)y}{1 + \alpha\gamma(1 - \delta)x(t - \tau_2)} - dy - Ey, \\ E(py - w) - m = 0, \end{cases} \tag{3}$$

where $r > 0$ is the intrinsic growth rate of prey; $K > 0$ is the carrying capacity of prey; d is the intrinsic mortality rate of the predator species. We assume the prey dynamics is delayed by τ_1 due to slow replacement of resources and the predator takes time τ_2 to convert the food into its growth. In the model, the economics of harvesting is described by the algebraic equation, where E is the predator-dependent harvesting rate, $p > 0$ is the harvesting reward, Ew is the total fixed cost and $m > 0$ is the fixed profit.

The initial conditions for the predator–prey subsystem (3) are

$$(x|_{[-\tau,0]}, y|_{[-\tau,0]}) \in C_+([-\tau, 0]; \mathcal{R}_+^2)$$

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