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Exponential stabilization of conservation systems with interior disturbance ☆

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ABSTRACT

In this paper, we consider the stabilization problem of the conservation systems with interior disturbance. Employing the idea of sliding-mode control, we design a nonlinear distributed feedback controller. We prove the solvability of the resulted closed-loop system by the maximal monotone operator theory. Further we prove the exponential stability of the closed-loop system. In particular, we prove that the requirement of a classical solution in the Lyapunov approach is unnecessary.

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1. Introduction

Let H be a Hilbert space and A be an unbounded, selfadjoint and positive definite linear operator with domain $D(A)$. We consider a second order linear system in H

$$\begin{cases} \ddot{w}(t) + Aw(t) = u(t) + r(t), \quad t > 0, \\ w(0) = w_0 \in D(A^{\frac{1}{2}}), \\ \dot{w}(0) = w_1 \in H \end{cases} \tag{1.1}$$

where $w(t)$ is a state of the system; $u(t)$ is a control and $r(t)$ is an interior disturbance. Usually the disturbance has a finite energy, that is, $\sup_{t\geq 0} \|r(t)\| < \infty$.

It is well-known that many practical systems are composed of certain flexible parts whose dynamic behavior is governed by hyperbolic partial differential equations (PDEs), such as wave equations, beam equations, plate equations, shells and so on, that are of the form (1.1). For these systems, one of the important tasks is to design feedback control laws to force the systems back to their equilibriums as fast as possible. In the past decades, there were many important stabilization results for the infinite dimensional systems without disturbance such as wave equation and flexible beam (e.g., see [4,14–16,34]), including

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stabilization results of the systems with time delay [29,30,32,35]. We observe that if these systems undergo the interior and exterior disturbances, which are of the form (1.1), the existing control laws will be invalid. Since the disturbances affect seriously the performance of the systems that may distort the systems, so we must redesign appropriate control laws to reject disturbances.

In the past decades, many scholars have been making great efforts for the anti-disturbance problems, for example, see [3,5,9,11–13,18–20,23,25–27,31,33]. One of the successful approaches of disturbance rejection is the sliding-mode control (briefly, SMC). For a long time, the sliding-mode control has been recognized as a powerful control tool [31] for the finite dimensional systems. In recent decades, scholars have been trying to extend this method to the infinite dimensional systems [18,23,25,26]. There exist some successful examples, for instance, Drakunov et al. in [9] proposed a sliding-mode control law for a heat equation with boundary control and disturbance; Cheng et al. in [5] studied a parabolic PDE system with parameter variations and boundary uncertainties by boundary sliding-mode control; Pisano et al. in [27] investigated a tracking control of wave equation with distributed control and disturbance, and applying the 2-SM they obtained the asymptotic stability of the closed-loop system. There are other approaches of anti-disturbance, such as active disturbance rejection control (ADRC) [12,19,33], Lyapunov control [11,13], adaptive control [6,36], and LMI-based design [10] etc. These methods also can be extended to the distributed parameter systems.

We observed that under the sliding-mode control design, the resulted system is a nonlinear system with discontinuous nonlinear term, the solvability and stability analysis of the closed-loop system are major difficulties. Under some sliding-mode control designs, the closed-loop systems might have no a solution in the sense of the classic solution. Therefore, when we design the feedback controller, we must take two things into account: stabilization property of the system and solvability of the closed-loop system.

In this paper, we mainly consider the stabilization problem of system (1.1). It is well-known that if there is no disturbance, then the system (1.1) becomes

$$\begin{cases} \ddot{w}(t) + Aw(t) = u(t), \quad t > 0, \\ w(0) = w_0, \quad \dot{w}(0) = w_1. \end{cases}$$

It has been proved that under the velocity feedback control

$$u(t) = -k\dot{w}(t), \quad k > 0,$$

the corresponding closed-loop system is exponentially stable [7,21]. However, due to the presence of $r(t)$, this stabilizer is not robust to the disturbance. To see this point, we consider the case that $r(t) = r$ is a constant. Under the feedback control law $u(t) = -k\dot{w}(t)$, the system (1.1) has a nonzero solution $(w_0, 0) = (A^{-1}r, 0)$. Therefore, we need to redesign control law to reject disturbance. Our idea for controller design is to divide the control into two parts: one ensures the exponential stability of the non-disturbance system and the other is used to reject disturbance. Employing an idea of the sliding-mode control, we take the feedback control law as

$$u(t) = -k\dot{w}(t) - M\frac{\dot{w}(t) + \rho w(t)}{||\dot{w}(t) + \rho w(t)||} \tag{1.2}$$

where the parameters ρ, k and M are positive constants with $\rho \in (0, k)$ and $M = \sup_{t\geq 0} ||r(t)||$. The relation between parameters k and ρ will be determined later.

Under the feedback control law (1.2), the closed-loop system associated with (1.1) is

$$\begin{cases} \ddot{w}(t) + Aw(t) = -k\dot{w}(t) - M\frac{\dot{w}(t)+\rho w(t)}{||\dot{w}(t)+\rho w(t)||} + r(t), \quad t > 0, \\ w(0) = w_0, \quad \dot{w}(0) = w_1. \end{cases} \tag{1.3}$$

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