J. Math. Anal. Appl. 436 (2016) 970–982

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Convergence rate estimates of solutions in a higher dimensional chemotaxis system with logistic source ☆

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A R T I C L E I N F O

Article history:
Received 23 May 2015
Available online 30 December 2015
Submitted by M. Winkler

Keywords:
Chemotaxis
Global existence
Large time behavior
Logistic source
Convergence rate

A B S T R A C T

We study the global attractors to the chemotaxis system with logistic source: $u_t - \Delta u + \chi\nabla\cdot(u\nabla v) = au - bu^2$, $\tau v_t - \Delta v = -v + u$ in $\Omega\times\mathbb{R}^+$, subject to the homogeneous Neumann boundary conditions, where smooth bounded domain $\Omega \subset \mathbb{R}^N$, with $\chi, b > 0$, $a \in \mathbb{R}$, and $\tau \in \{0, 1\}$. For the parabolic–elliptic case with $\tau = 0$ and $N > 3$, we obtain that the positive constant equilibrium $(\frac{a}{b}, \frac{a}{b})$ is a global attractor if $a > 0$ and $b > \max\{\frac{N-2}{N}\chi, \frac{\chi\sqrt{a}}{4}\}$. Under the assumption $N = 3$, it is proved that for either the parabolic–elliptic case with $\tau = 0$, $a > 0$, $b > \max\{\frac{\chi}{3}, \frac{\chi\sqrt{a}}{4}\}$, or the parabolic–parabolic case with $\tau = 1$, $a > 0$, $b > \frac{\chi\sqrt{a}}{4}$ large enough, the system admits the positive constant equilibrium $(\frac{a}{b}, \frac{a}{b})$ as a global attractor, while the trivial equilibrium $(0, 0)$ is a global attractor if $a \le 0$ and $b > 0$. It is pointed out that here the convergence rates are established for all of them. The results of the paper mainly rely on parabolic regularity theory and Lyapunov functionals carefully constructed.

1. Introduction

In this paper, we study the following Keller–Segel system with logistic source:

$$\begin{cases} u_t - \Delta u + \chi\nabla\cdot(u\nabla v) = au - bu^2, & x\in\Omega,\ t\in(0,T),\\ \tau v_t - \Delta v = -v + u, & x\in\Omega,\ t\in(0,T),\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & x\in\partial\Omega,\ t\in(0,T),\\ (u(x,0), \tau v(x,0)) = (u_0(x), \tau v_0(x)) \gneqq 0, & x\in\Omega, \end{cases} \tag{1.1}$$

☆ Supported by the National Natural Science Foundation of China (11171048, 11201046).
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http://dx.doi.org/10.1016/j.jmaa.2015.12.058
0022-247X/

where Ω is a bounded domain in $\mathbb{R}^N$ $(N \geq 3)$ with smooth boundary $\partial\Omega$, $0 < T \leq +\infty$, $\chi, b > 0$, $a \in \mathbb{R}$, $\tau \in \{0, 1\}$, and ν the outer normal vector on $\partial\Omega$. The initial data satisfy

$$u_0 \in C(\bar{\Omega}), \quad v_0 \in W^{1,q}(\Omega) \quad \text{for some } q > N, \tag{1.2}$$

compatible on $\partial\Omega$.

Eq. (1.1) is an extended version of the well-known Keller–Segel system with $a = b \equiv 0$ there, which was introduced by Keller and Segel [8] in 1970 to describe the cells (with density u) move towards the concentration gradient of a chemical substance v produced by the cells themselves. For the classical case the behavior of solutions depends on the interaction between the two mechanisms of diffusion and aggregation, and rich dynamic properties of solutions have been obtained such as the finite time blow-up [4,14,19,27], the global boundedness and large time behavior [3,15,25] of solutions. The global existence and large time behavior of solutions with the second equation replaced by another version modeling the consumption of the chemoattractant were considered in [21].

The model with logistic source was proposed by Mimura and Tsujikawa [13]. The influence of sources for the classical Keller–Segel system is significant, since the mass conservation is not true anymore with $a, b \not\equiv 0$. In general, the growth restriction from the logistic source would benefit the global existence and boundedness of solutions to the Keller–Segel models.

For $N \leq 2$, the boundedness and large time behavior of solutions of the model had been studied by Osaki et al. [17,18,16]. Some numerical results were shown in [2,5,20]. In this paper we consider the case of $N \geq 3$ to give the convergence rate estimates of solutions to the equilibrium with b appropriate large. It can be found that the methods of this paper is valid for the case of $N \leq 2$ as well.

For the parabolic–elliptic case with $\tau = 0$, Winkler [24] established the global existence of very weak solutions and gave the boundedness property under additional conditions that b is sufficiently large with $u_0 \in L^\infty(\Omega)$ having small norm in $L^\gamma(\Omega)$ for some $\gamma > \frac{N}{2}$. In [23], Tello and Winkler proved that if $b > \frac{N-2}{N}\chi$, then there exist globally bounded classical solutions to Eq. (1.1), and the equilibrium $(1, 1)$ is a global attractor if in addition $b > 2\chi$ and $a = b$. They also gave the global existence of weak solutions for arbitrary $b > 0$. With the self-diffusion of cells replaced by $\varepsilon\Delta u$, $\varepsilon \geq 0$ in (1.1), Winkler [29] and Lankeit [12] obtained that the carrying capacity $\frac{a}{b}$ can be exceeded to an arbitrary extent during evolution. More precisely, for $\chi = 1$, $a \geq 0$, with $N = 1$ and interval Ω [29], or $N \geq 2$ and ball Ω [12], if $b \geq 1$, then all solutions emanating from sufficient regular initial data are globally bounded in time; if $0 < b < 1$, then for each prescribed number $M > 0$, one can find some smooth initial data u_0 and small $\varepsilon > 0$ such that the solutions of (1.1) attain values above M at some time. Specially, if $\varepsilon = 0$ and $0 < b < 1$, there exist solutions blowing up in finite time.

For the parabolic–parabolic case with $\tau = 1$, Winkler [26] proved that if b is sufficiently large, there exist globally bounded classical solutions to problem (1.1). Moreover, the equilibrium $(\frac{1}{b}, \frac{1}{b})$ is a global attractor if $a = 1$ [28]. Under the assumption of Ω being convex, Lankeit [11] obtained that there exist global weak solutions even for arbitrary small values of $b > 0$, and if additionally $N = 3$ and $a \leq 0$, then $(0, 0)$ is a global attractor in $L^\infty(\Omega)$. In [22], Tao and Winkler proved that the population of cells always persists when $\chi, a, b > 0$, i.e., for any nonnegative global classical solution (u, v) of (1.1) (if it exists) with $u \not\equiv 0$, there is $m > 0$ such that

$$\int_\Omega u(x,t)dx \geq m \quad \text{for all } t > 0.$$

It was shown that for certain choices of the parameters, the solutions even may oscillate drastically in time [5].

Currently, global boundedness and large time behavior of solutions in a fully parabolic two-species chemotaxis system with competitive kinetics are completely solved by Bai and Winkler [1] for $N = 2$.