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The uniqueness of transonic shocks in supersonic flow past a 2-D wedge

Beixiang Fang[a], Wei Xiang[b,*]

[a] Department of Mathematics, and MOE-LSC, Shanghai Jiao Tong University, Shanghai 200240, China
[b] Department of Mathematics, City University of Hong Kong, Hong Kong, China

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ABSTRACT

We have proved the uniqueness of transonic shocks in steady supersonic flows past a slightly perturbed two-dimensional infinite wedge, under appropriate conditions on the downstream subsonic flows. We formulate it to a mathematical problem of the uniqueness of solutions of nonlinear partial differential equations of hyperbolic-elliptic mixed type with a free boundary. By working on several elliptic equations of physical quantities separately, we obtain *a priori* estimates of them, and then prove the uniqueness without assumptions on high regularity. Moreover, uniform estimates on the ellipticity and the positive lower bound of the speed are achieved under a geometrical condition on the wedge. The mathematical ideas and techniques developed here will also be useful for other related problems involving similar analytical difficulties.

Contents

* Corresponding author.
E-mail addresses: bxfang@sjtu.edu.cn (B. Fang), weixiang@cityu.edu.hk (W. Xiang).

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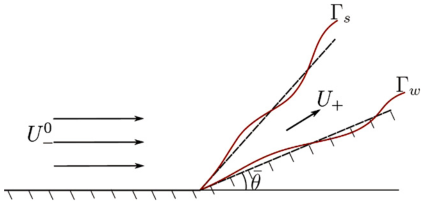

Fig. 1.1. The attached shock-front ahead of a wedge.

1. Introduction

1.1. Transonic shocks in supersonic flow past against a 2-D wedge

In this paper we study the uniqueness problem of transonic shocks in steady potential flow past a two-dimensional wedge. It is well-known that a shock-front appears when a supersonic flow past a straight wedge, see Fig. 1.1. In the case that the vertex angle of the wedge is less than some critical value, the shock-front attaches to the tip of the wedge. Moreover, for a given incoming supersonic flow, the flow field behind the shock-front can be determined by solving the Rankine–Hugoniot conditions. It has been shown, for instance in [6,12], that all admissible states behind the shock-front, which satisfies both R–H conditions and the entropy condition, form a loop on the velocity plane. The loop is called shock polar. A part of the shock polar is inside the sonic circle, thus the shock solutions are categorized as supersonic shocks—if the flow behind the shock-front is supersonic, and transonic shocks—if the flow behind the shock-front is subsonic. For a given wedge whose angle is less than the critical value depending on the incoming flow, there are two admissible shock solutions, within which the stronger one is always transonic, and the weaker one may be either supersonic or transonic.

It is a longstanding open problem in multidimensional conservation to pick out the right solution from these two entropy solutions. Courant and Friedrichs wrote in their classic monograph *Supersonic Flow and Shock Waves* [6, p. 317–318] that

> "······, then *two oblique shock fronts are possible* through which the flow is turned through the angle θ_K, a *weak* and a *strong* one. The question arises which of the two actually occurs. ······, the problem of determining which of the possible shocks occurs cannot be formulated and answered without taking the boundary conditions at infinity into account. ······ If the pressure prescribed there is below an appropriate limit, the weak shock occurs in the corner. If, however, the pressure at the downstream end is sufficiently high, a strong shock may be needed for adjustment. Under appropriate circumstances this strong shock may begin just in the corner and thus, of the two possibilities mentioned, the one giving a strong shock may actually occur.
>
> ······
>
> All statements made here are conjectures so far. While there is little doubt that they are in general correct, they should be supported, if possible, by detailed theoretical investigation."

Thanks to great efforts made by many mathematicians in the past decades, up to now, we already have deep understanding for the supersonic shock solutions, see, for instance, [1–3,5,11,13,15,17,18] and references therein. Compared with them, to our best knowledge, the mathematical theory for the transonic shocks is far away from satisfied. Some progress has been made for the two-dimensional steady flows in Chen–Fang [4], Fang [7], Yin–Zhou [16] and, recently, Fang–Liu–Yuan [8]. In particular, in [4,7,16], it was proved that the transonic shock is conditionally stable under perturbation of the upstream flow and/or perturbation of wedge boundary. In [8], it was proved that the piece-wise constant transonic shock solution is the unique solution when the wedge is straight provided appropriate conditions at the vertex and downstream at the