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Estimates for the spectrum on logarithmic interpolation spaces

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A B S T R A C T

We study spectral properties of operators on logarithmic perturbations of the real interpolation spaces with $\theta = 0$ or 1. We also establish estimates for the measure of non-compactness of operators interpolated by those methods.

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1. Introduction

The theory of the real interpolation method $(A_0, A_1)_{\theta,q}$ is well-established and has found important applications in function spaces and operator theory as it can be seen in the monographs by Bergh and Löfström [4], Triebel [27], König [22], Bennett and Sharpley [3] or Brudnyĭ and Krugljak [5]. Inside this theory, the spectral properties of operators on spaces $(A_0, A_1)_{\theta,q}$ for $0 < \theta < 1$ and $1 \le q \le \infty$ are one of the topics that have been investigated (see the papers by Zafran [28], Albrecht [2], Aksoy and Tylli [1] and the references given there).

Logarithmic perturbations of the real method $(A_0, A_1)_{\theta,q,\mathbb{A}}$ have been also extensively studied. The norm in $(A_0, A_1)_{\theta,q,\mathbb{A}}$ is

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$$\|a\,|(A_0,A_1)_{\theta,q,\mathbb{A}}\| = \Big(\int_0^\infty \big(t^{-\theta}\ell^{\mathbb{A}}(t)K(t,a)\big)^q\frac{dt}{t}\Big)^{1/q}$$

where K is the K-functional of Peetre, $1 \le q \le \infty$, $\mathbb{A} = (\alpha_0, \alpha_\infty) \in \mathbb{R}^2$, $\ell(t) = 1 + |\log t|$, $\ell^{\mathbb{A}}(t) = \ell^{\alpha_0}(t)$ if $0 < t \le 1$, $\ell^{\mathbb{A}}(t) = \ell^{\alpha_\infty}(t)$ if $1 < t < \infty$ and $0 \le \theta \le 1$. We refer to the papers by Evans and Opic [18], Evans Opic and Pick [19], Cobos and Segurado [14] and the references given there. The case $\theta = 1$ [respectively, $\theta = 0$] is of special interest since it produces spaces very close to A_1 [respectively, A_0]. Spaces $(A_0, A_1)_{1,q,\mathbb{A}}$, $(A_0, A_1)_{0,q,\mathbb{A}}$ are connected with limiting interpolation methods studied by Cobos, Fernández-Cabrera, Kühn and Ullrich [8] and Cobos and Kühn [13] among other authors. Logarithmic spaces with $\theta = 1, 0$ are useful in the work of Edmunds and Opic [17] on limiting variants of Krasnosel'skiĭ's compact interpolation theorem and in the subsequent abstract versions established by the present authors [11] and by Cobos and Segurado [14]. They also play a role in the work of Cobos and Domínguez [7] where they compare two kinds of Besov spaces with smoothness close to zero.

In this paper we continue the investigation on spaces $(A_0, A_1)_{1,q,\mathbb{A}}$ and $(A_0, A_1)_{0,q,\mathbb{A}}$. We show that if $A_0 \cap A_1$ is dense in A_0 and A_1 and $T \in \mathcal{L}((A_0, A_1), (A_0, A_1))$ with $T : A_1 \longrightarrow A_1$ being a Riesz operator, then $T : (A_0, A_1)_{1,q,\mathbb{A}} \longrightarrow (A_0, A_1)_{1,q,\mathbb{A}}$ is also a Riesz operator. Moreover, the spectrum of T on $(A_0, A_1)_{1,q,\mathbb{A}}$ coincides with the spectrum of T on A_1. We also establish the corresponding results for the case $\theta = 0$.

Those spectral properties are derived by using the estimates for the measure of non-compactness of interpolated operators that we show first and which are of independent interest. Let us recall that the behaviour under the real interpolation method of the measure of non-compactness was considered by Edmunds and Teixeira [26] assuming an additional approximation condition in the last couple, and by Cobos, Fernández-Martínez and Martínez [12] in the general case. Later some new ideas were shown by the present authors [9] where, among other things, they considered the case of logarithmic methods with $0 < \theta < 1$; previous results on that case are due to Cordeiro [15] and Szwedek [25]. Here we study the behaviour when $\theta = 1, 0$, obtaining results which apply to general Banach couples and work for the whole range of parameters. The special case when $\theta = 1$, $1 < q < \infty$, $\mathbb{A} = (-1, 0)$ and $A_1 \hookrightarrow A_0$, $B_1 \hookrightarrow B_0$ or $A_0 \hookrightarrow A_1$, $B_0 \hookrightarrow B_1$ has been already studied in [10].

The plan of the paper is as follows. In Section 2 we recall some basic results on logarithmic interpolation spaces and we establish norm estimates for interpolated operators. Section 3 deals with the measure of non-compactness and in the final Section 4 we derive the spectral properties of operators on spaces $(A_0, A_1)_{1,q,\mathbb{A}}$ and $(A_0, A_1)_{0,q,\mathbb{A}}$.

2. Logarithmic interpolation spaces

Let $\mathbb{A} = (\alpha_0, \alpha_\infty) \in \mathbb{R}^2$. Given a function $f : (0, \infty) \longrightarrow (0, \infty)$, we put

$$f^{\mathbb{A}}(t) = \begin{cases} (f(t))^{\alpha_0} & \text{if } 0 < t \le 1, \\ (f(t))^{\alpha_\infty} & \text{if } 1 < t < \infty. \end{cases}$$

We shall mainly work with the functions

$$\ell(t) = 1 + |\log t| \quad \text{and} \quad \ell\ell(t) = \ell(\ell(t)) = 1 + \log(1 + |\log t|).$$

Let $\bar{A} = (A_0, A_1)$ be a *Banach couple*, that is, two Banach spaces A_0, A_1 which are continuously embedded in some Hausdorff topological vector space. For $t > 0$, the *Peetre's K- and J-functionals* are given by

$$K(t, a) = K(t, a; A_0, A_1) = \inf\{\|a_0\,|A_0\| + t\|a_1\,|A_1\| : a_i \in A_i\},\ a \in A_0 + A_1\,,$$

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