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Convergence of a fluid–structure interaction problem decoupled by a Neumann control over a single time step

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ABSTRACT

Building off of previous analytical results for recasting fluid–structure interaction into an optimal control setting, an a priori error estimate is given for the optimality system by means of BRR theory. The convergence of the steepest descent method is proven in a discrete setting for a sufficiently small time step and mesh size. A numerical study is included supporting the theoretical rate of convergence over a single time step. Additional results demonstrate optimal convergence in space and time over several time steps.

1. Introduction

Fluid–structure interaction continues to be difficult to simulate numerically. In certain applications, particularly blood flow, the similar densities of the fluid and structure create a tight coupling that presents challenges in decomposing the problem and using existing solvers. This is most evident in the simulating of the flow of blood through a patient's artery. In order to correctly capture the flow, it is necessary to use a patient's geometry which requires three dimensional modeling. In order to reduce the computational workload needed, it is of particular importance that methods for solving FSI problems significantly reduce the number of iterations as well as the complexity of solvers at each time step.

A new *optimization-based approach* for solving fluid–structure interaction problems has recently been developed [20–22]. Although this method uses partitioned solvers, it is implicit and stable. It *avoids* the large number of iterations generally required in other partitioned methods, since a single control is used for both subsystems simultaneously, rather than iterating back and forth between subsystems. Previously, it has been shown that when applied to a Navier–Stokes/linear elastic fluid–structure interaction system, an optimal solution and Lagrange multipliers exist for a single time step [21,22].

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This optimization-based approach is a scheme for coupling possibly distinct fluid and structure solving codes together at a particular time step. The optimization problem should then be thought of as a partitioned approach for using the previous time step solution as data.[3,4]

In this work, we extend the analysis of the optimization based approach applied to the same fluid–structure system by rigorously proving the approximation error due to spatial discretization in Section 3.2 and the convergence of the steepest descent method in Section 3.3. In Section 4, the results of numerical simulations are presented, demonstrating that the theoretical rates of convergence are achieved.

2. Notation

We use the Sobolev spaces $W^{m,p}(D)$ with norms $\|\cdot\|_{m,p,D}$ if $p < \infty$, $\|\cdot\|_{m,\infty,D}$ if $p = \infty$. Denote the Sobolev space $W^{m,2}$ by H^m with the norm $\|\cdot\|_{m,D}$. The corresponding space of vector-valued or tensor-valued functions is denoted by $\mathbf{H}^m$.

For the variational formulation of the flow equations in the ALE framework, we define the function space for the reference domain:

$$\boldsymbol{H}^1_D(\Omega^f_{t_0}) := \{\mathbf{v} \in \mathbf{H}^1(\Omega^f_{t_0}) : \mathbf{v} = \mathbf{0} \text{ on } \Gamma^f_D\}.$$

The function spaces for Ω^f_t are then defined as

$$\boldsymbol{H}^1_D(\Omega^f_t) := \{\mathbf{v} : \Omega^f_t \times [t_0, T] \to \mathbb{R}^2,\ \mathbf{v} = \bar{\mathbf{v}} \circ \Psi_t^{-1} \text{ for } \bar{\mathbf{v}} \in \boldsymbol{H}^1_D(\Omega^f_{t_0})\},$$
$$L^2(\Omega^f_t) := \{q : \Omega^f_t \times [t_0, T] \to \mathbb{R},\ q = \bar{q} \circ \Psi_t^{-1} \text{ for } \bar{p} \in L^2(\Omega^f_{t_0})\},$$

where Ψ_t^{-1} is the inverse ALE mapping described later in this section.

For the structure displacement $\boldsymbol{\eta}$, define the function space

$$\boldsymbol{H}^1_D(\Omega^s) := \{\boldsymbol{\xi} \in \mathbf{H}^1(\Omega^s) : \boldsymbol{\xi} = \mathbf{0} \text{ on } \Gamma^s_D\}.$$

We use $(\cdot,\cdot)_{\Omega^f_t}$, $(\cdot,\cdot)_{\Gamma_{I_t}}$, $(\cdot,\cdot)_{\Omega^s}$, and $(\cdot,\cdot)_{\Gamma_{I_{t_0}}}$ to denote the L^2 inner product over Ω^f_t, Γ_{I_t}, Ω^s, and $\Gamma_{I_{t_0}}$, respectively.

In the moving fluid domain, we define the bilinear and trilinear forms

$$a(\mathbf{u},\mathbf{v})_{\Omega^f_t} = \frac{1}{4}\int_{\Omega^f_t} (\nabla\mathbf{u} + (\nabla\mathbf{u})^T) : (\nabla\mathbf{v} + (\nabla\mathbf{v})^T)\, d\Omega^f_t \quad \forall\, \mathbf{u},\mathbf{v} \in \boldsymbol{H}^1_D(\Omega^f_t),$$

$$b(\mathbf{v},q)_{\Omega^f_t} = -\int_{\Omega^f_t} q(\nabla\cdot\mathbf{v})\, d\Omega^f_t \quad \forall\, \mathbf{v} \in \boldsymbol{H}^1_D(\Omega^f_t),\ q \in L^2(\Omega^f_t),$$

and

$$c(\mathbf{u},\mathbf{v},\mathbf{w})_{\Omega^f_t} = \frac{1}{2}\int_{\Omega^f_t} \mathbf{u}\cdot\nabla\mathbf{v}\cdot\mathbf{w} - \mathbf{u}\cdot\nabla\mathbf{w}\cdot\mathbf{v}\ \ d\Omega^f_t.$$

[3] In practice, we have found that it is often not necessary to penalize the virtual control in the objective, which results in a sequence of solutions that are equivalent to the optimal control for the whole FSI problem posed as a global-in-time optimization problem, similar to [18].

[4] When the norm of the virtual control is included in the objective, the optimal control for the whole FSI problem posed as a global-in-time optimization problem is different from what will be achieved by this method piecing together a sequence of instantaneous controls. Solving time-dependent optimality systems by some iterative scheme that stores all state and adjoint solutions over all time steps is not practical or efficient for problems of this size. For more information about instantaneous control, see [8,16,24,25].