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Large deviation principle for stochastic Boussinesq equations driven by Lévy noise ☆

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ABSTRACT

The current paper is devoted to dynamics of stochastic Boussinesq equation driven by Lévy pure jump noise. The large deviation principle of solution is established, and the effect of the highly nonlinear and unbounded drift is stated. Moreover, the large deviation principle for stochastic two dimensional hydrodynamical systems with Lévy noise is also obtained.

1. Introduction

The Navier–Stokes equations are often coupled with other equations to describe a variety of phenomena in environmental, geophysical, and climate systems and so on. The Boussinesq equation is a coupled system of Navier–Stokes equations and the transport equations for temperature to describe the two-dimensional model for oceanic gravity currents [3]. There are many papers on the Boussinesq equation, we refer to [5,7,9,10,13] and so on, but there are a few papers on the stochastic Boussinesq equation driven by Gaussian noise, for example, the authors in [3] studied the random dynamics of the Boussinesq systems with dynamical boundary conditions, and showed the existence of random attractor. The authors in [8] showed the large deviations principle for the Boussinesq equations under random influences. The authors in [16] followed the idea of [3] and [8], and showed the large deviations principle for the Boussinesq equations with random dynamical boundary condition.

Stochastic partial differential equations (SPDE) driven by Lévy processes are widely applied in many fields such as financial market and so on, and attracted more and more attention recently [11,18–20]. D. Applebaum [2] introduced systematically the definition and relative properties of Lévy processes, and

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studies the fundamental theory of stochastic integrals such as the Wiener–Lévy-type stochastic integrals and Itô formula, and constructs the framework for the solution and Lévy flow of stochastic ordinary differential equations driven by Lévy processes. Recently, S. Peszat and J. Zabczyk constructed systematically general method on reproduct kernel Hilbert space for the existence, uniqueness and regularity of stochastic partial differential equations; we refer to [12] for the details.

There are a few papers about large deviation principle (LDP) for SPDE driven by Lévy noises in infinite dimensions, [14] may be the first one about LDP for SPDE with Lévy noise. It is necessary to point out that [14] just considered the evolution equation with Lipschitz coefficients. Recently, Xu and Zhang in [17] investigate the LDP for stochastic Navier–Stokes driven by additive Lévy noise, as they said in [17] that, the difficulties caused by Lévy noise are to deal with the highly nonlinear term $B(u, u)$. Stochastic Boussinesq equation with Lévy noise is more complicated, and it is harder to estimate the highly nonlinear terms $B_1(\cdot,\cdot)$ and $B_2(\cdot,\cdot)$ (the definition of B_1 and B_2 will be given in section 2).

Motivated by the idea of [17], we consider the LDP for stochastic Boussinesq equation driven by Lévy noises followed as

$$\begin{cases} \frac{\partial u^n}{\partial t} + (u^n\cdot\nabla)u^n - \nu\Delta u^n + \nabla p^n = \theta e_2 + b_1 dt + \frac{1}{\sqrt{n}}dW^1(t) + \int_X f(x)\tilde{N}_n^1(dt,dx), \\ \frac{\partial \theta^n}{\partial t} + (u^n\cdot\nabla)\theta^n - k\Delta\theta^n = u_2 + b_2 dt + \frac{1}{\sqrt{n}}dW^2(t) + \int_X g(x)\tilde{N}_2^n(dt,dx), \\ \nabla\cdot u^n = 0, \\ u^n|_{\partial D} = 0, \quad u^n(0) = u_0^n, \quad \theta^n(0) = \theta_0^n, \end{cases} \tag{1.1}$$

where velocity $u = u(x,t) = (u^1, u^2) \in R^2$, salinity $\theta = \theta(t,x) \in R$, pressure p, $x = (\xi,\eta) \in D \subset R^2$, $e_2 \in R^2$ is a unit vector in the upward vertical direction. $W^1(\cdot)$ and $W^2(\cdot)$ are H-valued Brownian motion, b_1 and b_2 are constants vector in H, f and g are measurable mappings from some measurable space X to H, $\tilde{N}_1^n$ and $\tilde{N}_2^n$ are compensated Poisson measure on $[0,\infty)\times X$ with intensity measure $n\nu_1$ an $n\nu_2$ respectively, where ν_1 and ν_2 are σ-finite measure on $\mathcal{B}(X)$, $f(x)$ and $g(x)$ satisfy

$$\int_U |f(x)|^2 e^{\alpha|f(x)|}\nu(dx) < \infty, \qquad \int_U |g(x)|^2 e^{\beta|g(x)|}\nu(dx) < \infty, \quad \forall\alpha > 0, \quad \forall\beta > 0. \tag{1.2}$$

It is necessary to point out that, Duan and Millet in [8] study the stochastic Boussinesq equations driven by Gaussian noises with periodic boundary condition. By using the weak convergence approach based on a variational representation of functionals of infinite dimensional Brownian motion, they show a large deviation principle. Recently, Chueshov and Millet [4] established the large deviation for a class of stochastic hydrodynamical systems with Gaussian white noise, including such as 2D-stochastic Navier–Stokes equation, 2D magneto-hydrodynamic equation, 2D Boussinesq model for the Bénard convection, 2D magnetic Bénard problem and 3D Leray α-model for Navier–Stokes equations.

Motivated by the works of Chueshov and Millet in [4], we will establish the large deviation principle for the stochastic hydrodynamical systems driven by Lévy noise, including 2D-stochastic Navier–Stokes equation, 2D magneto-hydrodynamic equation, 2D Boussinesq model for the Bénard convection, 2D magnetic Bénard problem, 3D Leray α-model for Navier–Stokes equations etc. Those stochastic models with Lévy noise can be represented uniformly as the following stochastic evolution equation

$$\begin{cases} du(t) + [Au(t) + B((u(t),u(t)) + R(u(t))]dt = bdt + \frac{1}{\sqrt{n}}dW^1(t) + \int_X f(x)\tilde{N}_n^1(dt,dx), \\ u(0) = u_0. \end{cases} \tag{1.3}$$

Under the appropriate assumptions, repeating the similar argument for stochastic Boussinesq equation (1.1), we can prove that the stochastic abstract equation (1.3) possesses the Large deviation principle, see section 5 for the details.