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Optimal boundary control of a steady-state heat transfer model accounting for radiative effects

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ABSTRACT

A boundary control problem for a nonlinear steady-state heat transfer model accounting for heat radiation effects is considered. The aim of control consists in obtaining a prescribed temperature distribution in a part of the model domain by controlling the boundary temperature. The solvability of this control problem is proven, and optimality conditions are derived. Numerical simulations are presented.

1. Introduction

Optimal control problems for models of complex heat transfer in scattering media with reflecting boundaries are of great importance in connection with engineering applications. A considerable number of works is devoted to problems of control of evolutionary systems describing radiative heat transfer (see [1–4,14–17]). In the mentioned works, the radiation transfer is described by an integro-differential equation or by its approximations. The temperature field is simulated by the conventional evolutionary heat transfer equation with additional source terms describing the contribution of the radiative heat transfer.

Among optimal control problems for steady-state models, it is worth to mention the work [12] where an optimal control problem for an elliptic semilinear PDE with nonlocal radiation interface conditions modeling conductive–radiative heat transfer was considered. This problem arises from the objective to optimize the temperature gradient during crystal growth by the physical vapor transport method. Theoretical analysis of

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optimal boundary control problems for steady-state systems of complex heat transfer is generally an open question. The main difficulty of such problems is related, apart from the nonlinear character of complex heat transfer equations in state variables, to nonlinearities in control inputs. In [8], a problem of optimal boundary multiplicative control for a steady-state complex heat transfer model was studied. The problem was formulated as the maximization of the energy outflow from the model domain by controlling reflection properties of the boundary. On the basis of new a priori estimates of solutions of the control system, the solvability of the optimal control problem was proven. The main result there was the proof of an analogue of the bang-bang principle arising in control theory for ordinary differential equations. Notice that there is an extensive literature on the bang-bang principle in optimal control of parabolic equations. In optimal control of elliptic equations, analogues of this principle are also available (see e.g. [11, Ch. 2, Remark 4.4]).

Problems considered in the present paper are formulated as minimization of a target functional by controlling the boundary temperature. The complexity of the theoretical analysis of such problems is caused by the following two features of the control system.

First, the control function, describing the boundary temperature, appears with power four in the boundary condition for the radiation intensity. Therefore, the proof of solvability of the optimal control problem is impossible without additional compactness assumptions on the set of admissible controls. It is reasonable, from the point of view of applications, to assume that the set of admissible controls has a finite dimension structure. In this case, the solvability of the control problem follows from continuity properties of the control-state mapping and the weak lower semicontinuity of the objective functional.

Second, the classical Lagrange principle cannot be applied to the derivation of necessary optimality conditions because the constraint operator is not affine with respect to the control. In the present paper, necessary optimality conditions are derived using the linearization of the complex heat transfer equations. The crucial point of the derivation is the requirement of unique solvability of the linearized system. The last question is nontrivial, and therefore sufficient conditions of unique solvability of the linearized and adjoint systems are formulated.

A numerical experiment of obtaining a desired temperature distribution in a part of the model domain by controlling the boundary temperature is conducted.

2. Problem formulation

The following steady-state normalized diffusion (P_1) model (see [7,9,10,13]) describing radiative, conductive, and convective heat transfer in a bounded domain $G \subset \mathbb{R}^3$ is under consideration:

$$-a\Delta\theta + \mathbf{v}\cdot\nabla\theta + b\kappa_a(|\theta|\theta^3 - \varphi) = 0, \quad -\alpha\Delta\varphi + \kappa_a(\varphi - |\theta|\theta^3) = 0, \tag{1}$$

$$a\partial_n\theta + \gamma(\theta - u)|_\Gamma = 0, \quad \alpha\partial_n\varphi + \beta(\varphi - u^4)|_\Gamma = 0. \tag{2}$$

Here, θ is the normalized temperature, φ the normalized radiation intensity averaged over all directions, $\mathbf{v}$ a given velocity field, and κ_a the absorption coefficient. The coefficients β and γ are given functions, and the others are given by

$$a = \frac{k}{\rho c_v}, \quad b = \frac{4\sigma n^2 T_{max}^3}{\rho c_v}, \quad \alpha = \frac{1}{3\kappa - A\kappa_s},$$

where k is the thermal conductivity, c_v the specific heat capacity, ρ the density, σ the Stefan–Boltzmann constant, n the refractive index, T_{max} the maximum temperature in the unnormalized model, $\kappa := \kappa_s+\kappa_a$ the extinction coefficient (total attenuation factor), and κ_s the scattering coefficient. The coefficient $A \in [-1, 1]$ describes the anisotropy of scattering. The symbol ∂_n denotes the derivative in the outward normal direction $\mathbf{n}$ on the boundary $\Gamma := \partial G$.