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Note

# Blowup of regular solutions for the relativistic Euler–Poisson equations

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ABSTRACT

In this paper, we study the blowup phenomena for the regular solutions of the isentropic relativistic Euler–Poisson equations with a vacuum state in spherical symmetry. Using a general family of testing functions, we obtain new blowup conditions for the relativistic Euler–Poisson equations. We also show that the proposed blowup conditions are valid regardless of the speed requirement, which was one of the key constraints stated in Geng (2015) [1].


## 1. Introduction

The isentropic relativistic Euler–Poisson equations [1] are expressed as follows:

$$\begin{cases} \partial_t(\frac{n}{\sqrt{1-|v|^2/c^2}}) + \nabla\cdot(\frac{nv}{\sqrt{1-|v|^2/c^2}}) = 0, \\ \partial_t(\frac{p/c^2+\rho}{1-|v|^2/c^2}v) + \nabla\cdot(\frac{p/c^2+\rho}{1-|v|^2/c^2}v\otimes v) + \nabla p = \frac{4\pi n\nabla\phi}{\sqrt{1-|v|^2/c^2}}, \\ \Delta\phi = \frac{4\pi n}{\sqrt{1-|v|^2/c^2}}, \end{cases} \tag{1}$$

where $n$ is defined by

$$\rho = n\big(1+e/c^2\big), \tag{2}$$

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which satisfies

$$\frac{\mathrm{d}n}{nc^2} = \frac{\mathrm{d}\rho}{p + \rho c^2}. \tag{3}$$

The unknowns and constants in the above equation are defined as follows: $\rho : [0, \infty) \times \mathbf{R}^3 \to [0, \infty)$ and $n : [0, \infty) \times \mathbf{R}^3 \to [0, \infty)$ denote the proper mass-energy density and the charge density, respectively; $c$ is the speed of light; $v : [0, \infty) \times \mathbf{R}^3 \to \mathbf{R}^3$ is the velocity of an electro-fluid; $-\phi : [0, \infty) \times \mathbf{R}^3 \to \mathbf{R}$ is the electrostatic potential in the inertial frame; and $p = p(\rho)$ is the pressure function of the electro-fluid in a proper frame. The equation of state $p$ follows the $\gamma$-law:

$$p = \rho^\gamma, \tag{4}$$

where $\gamma > 1$ is the adiabatic index, and the speed of sound is

$$\sqrt{p'(\rho)} < c, \tag{5}$$

where $c$ is the speed of light which is the highest speed in the special relativity.

Lastly, the constant $e \geq 0$ in (2) is the specific internal energy.

Relativistic electrodynamics includes the study of the interaction between relativistic charged particles and electromagnetic fields when the particles are moving at a speed comparable to the speed of light in a vacuum. At such a high speed, the motion of the charged particles no longer obeys the Newtonian equations, so relativistic equations of particles must be applied. Under a field with a much stronger electric than magnetic effect, such as those in supernova explosions, gravitational collapse, and the formation and expansion of black holes and neutron stars, the motion of an isentropic relativistic electro-fluid can be described by the Euler–Poisson equations (2) when the charged particles are moving very fast.

To understand the mathematical nature of relativistic fluid dynamics, we first review a related and previously developed relativistic model, namely, the relativistic Euler equations. Makino and Ukai [8,9] and LeFloch and Ukai [5] established the local existence of classical solutions to the relativistic system using the theory of a quasi-linear symmetric hyperbolic system. More precisely, the critical part of Makino and Ukai's proof was based on the existence of a strictly convex entropy for the non-vacuum case; the critical part of LeFloch and Ukai's proof relied on the generalized Riemann invariants and normalized velocity for the vacuum case. Geng and Li [2] extended these results to the isentropic system. For the non-isentropic system, Guo and Tahvildar-Zadeh [4] proved the blowup result for smooth solutions using the averaged quantities method developed by Sideris [13,14]. Moreover, Pan and Smoller [11] applied the classical energy method to show the singularity formation of smooth solutions.

Due to the complexity of the structures of system (1), research on multi-dimensional relativistic Euler–Poisson equations is still at an early stage. In 2013, Mai, Li and Zhang [7] gave the first well-posed result for the steady-state relativistic Euler–Poisson equations with relaxation. For system (1) in the one dimensional case, Geng and Wang [3] obtained the global existence of a smooth solution with some monotonic conditions on the initial data. The importance of system (1) is that the non-relativistic Euler–Poisson equations are the Newtonian limit of system (1). Readers may refer to [6,12,18,19,15] for the blowup results of the non-relativistic Euler–Poisson equations.

In this paper, we consider the spherical symmetric solutions, namely,

$$n = n(t, r), \qquad \rho = \rho(t, r), \qquad v = \frac{x}{r} v(t, r), \tag{6}$$

where $r = |x|$ is the radius of the spatial variables $x \in \mathbf{R}^3$.