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Flux-approximation limits of solutions to the relativistic Euler equations for polytropic gas ☆

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ABSTRACT

The flux-approximation problem of the relativistic Euler equations for polytropic gas in special relativity is studied. At first, we solve the Riemann problem of the pressureless relativistic Euler equations with a flux approximation, and obtain two kinds of solutions involving a family of delta shock wave and pseudo-vacuum state. Then, as the flux approximation vanishes, we show that the limits of the family of delta-shock and pseudo-vacuum solutions are exactly the delta-shock and vacuum state solutions of the pressureless relativistic Euler equations, respectively. Next, the Riemann problem of the relativistic Euler equations with a double parameter flux approximation including pressure is solved analytically. Furthermore, it is rigorously proved that, as the double parameter flux perturbation vanishes, any two-shock Riemann solution tends to a delta-shock solution to the pressureless relativistic Euler equations; any two-rarefaction Riemann solution tends to a two-contact-discontinuity solution to the pressureless relativistic Euler equations and the nonvacuum intermediate state in between tends to a vacuum state.

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1. Introduction

The well-known Euler system of conservation laws of energy and momentum in special relativity reads

$$\begin{cases} \left((p+\rho c^2)\dfrac{v^2}{c^2(c^2-v^2)}+\rho\right)_t + \left((p+\rho c^2)\dfrac{v}{c^2-v^2}\right)_x = 0, \\ \left((p+\rho c^2)\dfrac{v}{c^2-v^2}\right)_t + \left((p+\rho c^2)\dfrac{v^2}{c^2-v^2}+p\right)_x = 0, \end{cases} \tag{1.1}$$

where ρ, v and p represent the proper energy density, particle speed and pressure, c is the speed of light, and the physically relevant region for solution is $\{(\rho, v) \mid \rho \geq 0, |v| < c\}$. The system (1.1) models the dynamics of plane waves in special relativity fluids, see [19–22] in a two-dimensional Minkowski space–time (x^0, x^1)

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$$\text{div}\, T = 0,$$

with the stress-energy tensor for a fluid

$$T^{ij} = (p + \rho c^2)u^i u^j + p\eta^{ij},$$

where all indices run from 0 to 1 with $x^0 = ct$, $\eta^{ij} = \eta_{ij} = diag(-1, 1)$ denotes the flat Minkowski metric, u the 2-velocity of the fluid particle, and ρ the mass-energy density of the fluid as measured in units of mass in a frame moving with the fluid particle.

In general, the solution to the system (1.1) strongly depends on the state equation $p = p(\rho)$. In this paper, we are concerned with the polytropic gas, whose state equation can be formulated as

$$p(\rho) = \kappa^2 \rho^\gamma, \qquad \gamma > 1, \tag{1.2}$$

where κ is a positive constant satisfying $\kappa < c$. When $\gamma = 1$, (1.2) models an isothermal gas, which corresponds to the extremely relativistic gases, when the temperature is very high and the particles move near the speed of light. This case will be studied in the future.

The study of the relativistic Euler equations (1.1) has been attracting more and more challenging attention of mathematics and physics researchers due to its importance and extreme complexity. Smoller and Temple [16] studied the Riemann problem and Cauchy problem of the system (1.1) when $\gamma = 1$. While for the case $\gamma > 1$, Chen [4] analyzed the properties of elementary waves, and solved the Riemann problem and Cauchy problem. Further, Chen and Li [1] established the uniqueness of Riemann solutions in the class of entropy solutions with arbitrarily large oscillation. Li, Feng and Wang [13] established the global existence of the entropy solutions with a class of large initial data which involve the interaction of shock waves and rarefaction waves. Recently, Ding and Li [6] studied a kind of multidimensional piston problem for (1.1). They established the local existence of shock front solutions to the spherically symmetric piston problem, as well as the convergence of the local solution as $c \to \infty$ to the corresponding solution of the classical non-relativistic Euler equations. Cheng and Yang [5] solved the Riemann problem of (1.1) for the Chaplygin gas.

As the pressure vanishes, that is $\kappa \to 0$, the limit system of (1.1) formally becomes the following pressureless relativistic Euler equations

$$\begin{cases} \left(\dfrac{\rho}{c^2 - v^2}\right)_t + \left(\dfrac{\rho v}{c^2 - v^2}\right)_x = 0, \\ \left(\dfrac{\rho v}{c^2 - v^2}\right)_t + \left(\dfrac{\rho v^2}{c^2 - v^2}\right)_x = 0, \end{cases} \tag{1.3}$$

which are fully linearly degenerate. The classical elementary waves only involve contact discontinuities. Interestingly, delta shock waves and vacuum states do occur in solutions. As for the delta shock waves, there have been rich results for various strictly or nonstrictly hyperbolic systems of conservation laws, see [7–10,12,14,15,17,18,23,27,28] and the reference cited therein.

In the past decade, the vanishing pressure limit method has been introduced to explore the phenomena of concentration and cavitation and the formation of delta shock waves and vacuum states in solutions, say Li [11] for the compressible Euler equations with zero temperature, Chen and Liu [2,3] for the isentropic and nonisentropic fluids. Recently, Yin and Sheng [29,30] for the system (1.1) and the relativistic fluid dynamics, Yin and Song [31] for the Chaplygin gas, and Yang and Wang [26] for the modified Chaplygin gas, etc. It is noticed that all these works on this topic are only focused on the pressure level.

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