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Invertibility of Fock Toeplitz operators with positive symbols ☆

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ABSTRACT

In this paper, we characterize the invertibility of Fock Toeplitz operators with positive symbols via their Berezin transforms and the reverse Carleson measure for Fock space.

1. Introduction

Let $d\mu$ be the Gaussian measure on the complex plane $\mathbb{C}$. It is well known that, in terms of the standard area measure $dm = \frac{1}{\pi}dxdy$ on $\mathbb{C}$, we have

$$d\mu(z) = \frac{1}{2}e^{-\frac{|z|^2}{2}}dm(z).$$

Recall that the Fock space $\mathcal{F}^2$ is defined to be the subspace

$$\Big\{f \text{ is analytic on } \mathbb{C} : \|f\|^2 := \int_{\mathbb{C}} |f(z)|^2 d\mu(z) < +\infty\Big\}.$$

It can be easily checked that $\mathcal{F}^2$ is a reproducing kernel Hilbert space. As usual, let k_z denote the normalized reproducing kernel for $\mathcal{F}^2$. That is,

$$k_z(w) = e^{\frac{\bar{z}w}{2} - \frac{|z|^2}{4}}.$$

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Let $P : L^2(\mathbb{C}, d\mu) \to \mathcal{F}^2$ be the orthogonal projection, then for each φ in $L^\infty(\mathbb{C})$, one can define the Fock Toeplitz operator and Fock Hankel operator with symbol φ as follows:

$$T_\varphi f = P(\varphi f)$$

and

$$H_\varphi f = (I - P)(\varphi f).$$

For more information on topics of Toeplitz operators and Hankel operators on the Fock space we refer to [7] and [8].

A positive Borel measure ν will be called a Fock Carleson measure if there is a constant $C > 0$ such that

$$\int_{\mathbb{C}} |f(z)|^2 e^{-\frac{|z|^2}{2}} d\nu(z) \leqslant C \int_{\mathbb{C}} |f(z)|^2 e^{-\frac{|z|^2}{2}} dm(z) = 2C \int_{\mathbb{C}} |f(z)|^2 d\mu(z)$$

for all $f \in \mathcal{F}^2$, see [1] and [8] if needed. Likewise, we can define the reverse Fock Carleson measure. It is a positive Borel measure ν such that the reverse Carleson inequality holds, i.e., there exists a constant $C > 0$ such that the following inequality holds for all $f \in \mathcal{F}^2$:

$$\int_{\mathbb{C}} |f(z)|^2 e^{-\frac{|z|^2}{2}} d\nu(z) \geqslant C \int_{\mathbb{C}} |f(z)|^2 e^{-\frac{|z|^2}{2}} dm(z).$$

Given a subset G in the unit disk $\mathbb{D}$, let χ_G be the characteristic function of G. Luecking completely characterized the properties of G in order that the measure $\chi_G(z)dm(z)$ to be a reverse Bergman Carleson measure and several necessary and sufficient conditions on G were established in [2]. Moreover, he extended these results to several variables and to larger classes of domains [3]. For more interesting results on this topic we refer to his another paper [4].

Recently, using the reverse Bergman Carleson measure and Luecking's results in [2], the second author and D. Zheng characterized the invertibility of Bergman Toeplitz operators with positive symbols by their Berezin transforms [6]. Indeed, they proved that the Bergman Toeplitz operator with positive symbol is invertible if and only if its Berezin transform is invertible in $L^\infty(\mathbb{D})$. However, it is difficult to study the invertibility of Bergman Toeplitz operators in the general case, even if the symbols are harmonic on $\mathbb{D}$.

Motivated by the results in [2] and [6], we consider the analogous questions in the case of Fock space. Furthermore, we characterize the invertibility of the Fock Toeplitz operators with positive symbols by their Berezin transforms and the reverse Fock Carleson measure. In particular, we obtain a necessary and sufficient condition on $G \subset \mathbb{C}$ such that $\chi_G dm$ is a reverse Fock Carleson measure, which is parallel to the main theorem in [2], see Theorem 1.2 below.

The main results of this paper are the following.

Theorem 1.1. *Let $\varphi \in L^\infty(\mathbb{C})$ and $\varphi \geqslant 0$ a.e. Then the following are equivalent:*

(1) The Toeplitz operator T_φ is invertible on $\mathcal{F}^2$;

(2) There exists $r > 0$ such that the set

$$\{z \in \mathbb{C} : \varphi(z) > r\}$$

satisfies Condition (i) in Theorem 1.2;

(3) The Berezin transform $\widetilde{\varphi}$ is invertible in $L^\infty(\mathbb{C})$;

(4) There exists a constant $C > 0$ such that