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Journal of Mathematical Analysis and Applications

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# Uniqueness of large positive solutions for a class of radially symmetric cooperative systems $\stackrel{\stackrel{}_{\leftrightarrow}}{\sim}$

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#### ARTICLE INFO

Article history: Received 5 June 2015 Available online 17 November 2015 Submitted by V. Radulescu

Keywords: Large positive solution Cooperative system Radial symmetry Maximum principle Uniqueness

#### 1. Introduction

This paper studies the existence and uniqueness of the solution of the, eventually singular, elliptic problem

$$\begin{cases} -\Delta u_i = \lambda_i u_i + \sum_{\substack{j=1, j \neq i}}^n a_{ij} u_j - \alpha_i(d(x)) f_i(u_i) u_i & \text{in } \Omega, \\ u_i = M & \text{on } \partial\Omega, \end{cases} \quad 1 \le i \le n, \tag{1.1}$$

where  $M \in (0, \infty]$ ,  $\lambda_i$ ,  $a_{ij} \in \mathbb{R}$  for all  $1 \le i \le j \le n$ ,  $j \ne i$ ,  $n \in \mathbb{N}$ , with  $a_{ij} > 0$ , and

$$d(x) := \operatorname{dist}(x, \partial \Omega), \qquad x \in \Omega,$$

is the distance to the boundary function. The kind of domains  $\Omega$  considered here are the ball and the annulus. So,

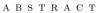
$$\Omega \in \{B_R(x_0), A_{R_1, R_2}(x_0)\},\tag{1.2}$$

http://dx.doi.org/10.1016/j.jmaa.2015.11.026 0022-247X/© 2015 Elsevier Inc. All rights reserved.









This paper shows the uniqueness of the large solution of a class of radially symmetric sublinear elliptic systems of cooperative type. This result is based on the strong maximum principle, rather than on the blow-up rates of the large solutions, as it is common in most the available results for single equations.

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 <sup>&</sup>lt;sup>\*</sup> Supported by the Ministry of Economy and Competitiveness under Research Grants MTM2012-30669 and MTM2015-65899-P.
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where  $x_0 \in \mathbb{R}^N$ ,  $N \ge 1$ , R > 0,  $R_2 > R_1 > 0$ , and

$$B_R(x_0) := \{ x \in \mathbb{R}^N : |x - x_0| < R \},\$$
$$A_{R_1, R_2}(x_0) := \{ x \in \mathbb{R}^N : R_1 < |x - x_0| < R_2 \}$$

In (1.1), for every  $1 \le i \le n$ , we suppose that  $\alpha_i \in \mathcal{C}^{\nu}[0,\infty)$ , for some  $\nu \in (0,1]$ , satisfy  $\alpha_i \ge 0$  in  $\Omega$  and  $\alpha := (\alpha_1, \dots, \alpha_n) \ne 0$ . As far as concerns the nonlinear terms of (1.1), the following conditions are imposed:

- (A1) For every  $1 \le i \le n$ ,  $f_i \in \mathcal{C}^{1+\nu}[0,\infty)$ ,  $f_i(0) = 0$  and  $f'_i(u) > 0$  for all u > 0.
- (A2) There exists  $F \in \mathcal{C}^{1+\nu}[0,\infty)$  such that F(0) = 0, F(u) > 0, F'(u) > 0 for all u > 0, and

$$\min_{1 \le i \le n} f_i(u) \ge F(u) \quad \text{for all } u \ge 0, \qquad \lim_{u \uparrow \infty} F(u) = \infty.$$

In the singular case when  $M = \infty$ , a function

$$u = (u_1, \cdots, u_n) \in [\mathcal{C}^{2+\nu}(\Omega)]^n$$

is a solution of (1.1) if it satisfies the system and

$$\lim_{d(x)\downarrow 0} u_i(x) = \infty \quad \text{for all } 1 \le i \le n$$

In such case, u is said to be a *large*, or explosive, solution of (1.1).

Under these general hypotheses, the problem (1.1) has a unique positive solution for every  $0 < M < \infty$ , denoted by  $\theta_{[\lambda,\Omega,M]}$  with  $\lambda := (\lambda_1, \ldots, \lambda_n)$ . Moreover, by the maximum principle,  $M \mapsto \theta_{[\lambda,\Omega,M]}$  is point-wise increasing. Thus, the limiting function

$$\theta_{[\lambda,\Omega,\infty]} := \lim_{M \uparrow \infty} \theta_{[\lambda,\Omega,M]} \tag{1.3}$$

is well defined, though it might equal  $\infty$  somewhere in  $\Omega$ , unless the following generalized Keller–Osserman condition, [12,28], holds

(KO) Assumptions (A1) and (A2) hold and, for every  $\beta > 0$ ,  $\lim_{u \uparrow \infty} I(u) = 0$ , where

$$I(u) := \int_{1}^{\infty} \frac{d\theta}{\sqrt{\int_{1}^{\theta} [\beta F(ut) - 1] t dt}}.$$

See [19, Chapter 3] for a detailed discussion about condition (KO). In particular, under the assumption (A1), the condition (KO) holds if there are p > 0 and C > 0 such that

$$f_i(u) \ge F(u) := Cu^p \quad \text{for all } u \ge 0 \quad \text{and} \quad 1 \le i \le n.$$
 (1.4)

The main result of this paper is the following

**Theorem 1.1.** Suppose (KO),  $\Omega \in \{B_R(x_0), A_{R_1,R_2}(x_0)\}, \lambda_i \geq 0, a_{ij} > 0$  for all  $1 \leq i \leq j \leq n, j \neq i, \alpha_i$  are positive nondecreasing functions,

$$0 < \alpha_i(t) \le \alpha_i(s) \quad \text{for all} \quad 0 < t \le s, \quad 1 \le i \le n,$$

$$(1.5)$$

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