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Stability analysis for stochastic differential equations with infinite Markovian switchings

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ABSTRACT

This paper is concerned with stability analysis of linear Itô stochastic differential equations with countably infinite Markovian switchings. A spectral criterion is proposed for exponential stability of the considered models. By means of the established spectral criterion, the relationship between exponential stability and stochastic (L_2) stability is clarified. Moreover, under the disturbance of random signals with finite energy, a sufficient condition is presented for L_2 input–output stability of the perturbed stochastic differential equations.

1. Introduction

Stochastic differential equations (SDEs) with Markovian switchings have received considerable attention in the past decades [5,13]. There have been fruitful results contributed to the issues about this type of models, ranging from stability [11,12] and structural analysis [15] to scientific and engineering applications [16,20]. It should be pointed out that most of the existing works are concentrated on the case of finite Markovian switchings. However, in many real scenarios, it may be more appropriate to describe the structural changes of dynamical plants by a Markov chain with infinite states [3]. Therefore, some researchers have turned to study stochastic differential or difference equations with Markov process taking values in an infinite set [2,14,17]. Particularly, as one of the central subjects in the equation theory, stability analysis has attracted an increasing interest; see [4,8,10,18] and the references therein for the latest developments. It has been found that SDEs with infinite Markovian switchings possess some distinct features; e.g., asymptotic mean square stability and stochastic stability (L_2 stability) are equivalent for SDEs with finite Markovian switchings, while this link does no more hold in the case of infinite Markovian switchings [7].

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In this paper, we will focus on the stability analysis of linear Itô SDEs with countably infinite Markovian switchings. First of all, a spectral criterion will be established for exponential (mean square) stability of the considered equations. Based on the obtained criterion, the intrinsic relationship between exponential stability and stochastic stability will be revealed. It will be shown that, different from the SDEs with finite Markovian switchings, even if the SDE with infinite Markovian switchings is stochastically stable, it may be not exponentially stable. Moreover, for any finite-energy random disturbance, it is exponential stability, instead of stochastic stability, that can guarantee the perturbed SDEs with infinite Markovian switchings to be L_2 input–output stable.

The remainder of this paper is organized as follows. Section 2 provides some useful preliminaries. In Section 3, a spectral criterion and a Barbasin–Krasovskii-type theorem are presented for exponential stability. Section 3.2 proceeds with the analysis of L_2 input–output stability. Finally, Section 4 ends this paper with a concluding remark.

Notations. $\mathcal{C}$ (R): the set of all complex (real) numbers; $\mathcal{C}^-$: the open left-hand side complex plane; R_+: the set of all nonnegative real numbers; R^n: n-dimensional space with the usual Euclidean norm $\|\cdot\|$; S_n: the set of all $n\times n$ symmetric matrices, whose entries may be complex; $A>0$ (≥ 0): A ($\in S_n$) is positive (semi-positive) definite; A': the transpose of a matrix (vector) A; $\Lambda(\cdot)$: the eigenvalue set of some square matrix or operator; I_n: the n-dimensional identity matrix; $\delta(\cdot)$: the Kronecker delta function.

2. Preliminaries

On a complete probability space $(\Omega,\mathcal{F},P)$, we consider the following linear SDE with Markovian switchings:

$$\begin{cases} dx(t)=A_0(r(t))x(t)dt+\sum\limits_{k=1}^{l}A_k(r(t))x(t)dw_k(t), \\ dz(t)=C_0(r(t))x(t)+\sum\limits_{k=1}^{l}C_k(r(t))x(t)dw_k(t),\ t\in R_+, \end{cases} \tag{1}$$

where $x(t)\in R^n$ and $z(t)\in R^{n_z}$ represent the state and measurement output, respectively. Let $w(t)=(w_1(t)\cdots w_l(t))'$ be a standard l-dimensional Brownian motion defined on the given probability space. In (1), all coefficients are real matrices of appropriate dimensions, e.g., $A_0(i)\in R^{n\times n}$. The stochastic process $\{r(t)\}_{t\in R_+}$ is a Homogeneous Markov chain which is right continuous and takes values in the countably infinite set $\mathbb{Z}_+=\{1,2,\cdots\}$. The transition probability matrix of $\{r(t)\}_{t\in R_+}$ is denoted by $P(t)=(p_{ij}(t))_{i,j\in\mathbb{Z}_+}$ with $p_{ij}(t)=P\{r(t+h)=j|r(h)=i\}$ for $t>0$ and $h\geq 0$. The infinitesimal matrix of $\{r(t)\}_{t\in R_+}$ is $\Gamma=(q_{ij})_{i,j\in\mathbb{Z}_+}$, where

$$q_{ij}=\lim_{t\searrow 0}\frac{p_{ij}(t)-p_{ij}(0)}{t} \tag{2}$$

and $p_{ij}(0)=\delta(i-j)$. It is assumed that $q_{ij}\geq 0$ for $i\neq j$, $0\leq -q_{ii}=\sum_{j\neq i}q_{ij}<c$ ($i,j\in\mathbb{Z}_+$) for some $c>0$ and $\sup\limits_{i\in\mathbb{Z}_+}\sum_{j=1}^{\infty}|q_{ij}|<\infty$. The initial state $x(0)\in R^n$ is deterministic, and the initial distribution of Markov process satisfies $\pi_i(0):=P(r(0)=i)>0$ ($i\in\mathbb{Z}_+$). For arbitrary $t\in R_+$, the σ-algebras $\mathcal{G}_t:=\sigma(w(s),0\leq s\leq t)$ and $\mathcal{H}_t:=\sigma(r(s),0\leq s\leq t)$ are mutually independent. Moreover, $\hat{\mathcal{H}}_t$ ($t\geq 0$) is the smallest σ-algebra that contains all sets $M\in\mathcal{F}$ with $P(M)=0$ and $w(s)$ ($s\leq t$) is measurable with respect to $\hat{\mathcal{H}}_t$. Denote by $\mathcal{F}_t:=\mathcal{G}_t\vee\mathcal{H}_t\vee\hat{\mathcal{H}}_t$. Let $L^2_{\mathcal{F}}(R_+,R^{n_y})$ be the space of all nonanticipative stochastic processes $y(t)\in R^{n_y}$ (with respect to $\mathcal{F}_t$) satisfying

$$\|y\|_{L^2_{\mathcal{F}}(R_+)}:=\left\{E\int_0^\infty\|y(t)\|^2dt\right\}^{1/2}<\infty.$$