J. Math. Anal. Appl. 438 (2016) 856–874

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Large time behavior of solutions to multi-dimensional bipolar hydrodynamic model of semiconductors with vacuum

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A R T I C L E   I N F O

Article history:
Received 28 July 2015
Available online 8 February 2016
Submitted by D. Wang

Keywords:
Euler–Poisson systems
Bipolar multi-dimensional semiconductor device
Stationary solution
Entropy solution
Large time behavior

A B S T R A C T

In this paper, a multi-dimensional hydrodynamic model for the bipolar semiconductor device with insulating boundary conditions and a non-flat doping profile is investigated. The model is assumed to be spherically symmetrical. A large time behavior framework is builded for any L^∞ bounded radial symmetry weak solutions, that is, the weak entropy solutions are shown to converge to the corresponding stationary solutions in L^2 norm and an exponential decay rate is also derived. No smallness and regularity conditions are assumed and the doping profile are permitted to be of large variation.

1. Introduction

In this paper, we consider the following Euler–Poisson system for the bipolar hydrodynamical model of semiconductor device

$$\begin{cases} \partial_t \rho_1 + \nabla \cdot \vec{m}_1 = 0, \\ \partial_t \vec{m}_1 + \nabla \cdot \big(\dfrac{\vec{m}_1 \otimes \vec{m}_1}{\rho_1}\big) + \nabla p(\rho_1) = \rho_1 \nabla \phi - \vec{m}_1, \\ \partial_t \rho_2 + \nabla \cdot \vec{m}_2 = 0, \\ \partial_t \vec{m}_2 + \nabla \cdot \big(\dfrac{\vec{m}_2 \otimes \vec{m}_2}{\rho_2}\big) + \nabla q(\rho_2) = -\rho_2 \nabla \phi - \vec{m}_2, \\ \Delta \phi = \rho_1 - \rho_2 - D(\vec{x}), \end{cases} \tag{1.1}$$

where $\vec{x} \in \mathbf{R}^{\mathbf{N}} (\mathbf{N} \geq 1)$ is the space variable, $t \in \mathbf{R}_+ = [0, \infty)$ is the time variable. ρ_1, ρ_2, $\vec{m}_1$, $\vec{m}_2$, ϕ, $\nabla\phi$ are the unknown functions of $\vec{x}$ and t, representing the electron density, the hole density, the electron current density, the hole current density, the potential function and the electric field respectively. The pressure

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http://dx.doi.org/10.1016/j.jmaa.2016.01.051
0022-247X/

terms $p(\rho_1)$ and $q(\rho_2)$ denote the pressure-density relations. And the function $D(\vec{x}) > 0$, called the doping profile, stands for the density of impurities in semiconductor devices. The symbols $(\nabla\cdot)$ and Δ denote the divergence and Laplacian in $\mathbf{R}^{\mathbf{N}}$, the symbol $\bigotimes$ denotes the Kronecker tensor product. Several physical constants have been set to unity for the simplicity of presentation.

The text books [16] and [23] are good references for the derivation of the hydrodynamic model of semiconductors. It is worthy to point out that the studies on the Euler-Poisson system for the unipolar hydrodynamic model (i.e., $\rho_2 = 0$, $\vec{m}_2 = \vec{0}$ in (1.1)), such as the studies on the existence of smooth or weak solutions and their large time behavior as well as relaxation-time limit, have been extensively carried out, see [3,5,6,9–12,18,20–22,26,28,29,31] etc. Due to the complexity of the system and other technical difficulties, the related study on the bipolar semiconductor models is very limited so far, see [7,8,13,14,17,19,24,25,32] and the references therein.

In this note, we consider the isothermal fluids, that is, the pressure terms are assumed to be $p(\rho_1) = \rho_1$, $q(\rho_2) = \rho_2$, which is a real physical case. Moreover, for conciseness and from physical motivation, we focus on the spacial domain $\Omega = \{\vec{x} \in \mathbf{R}^{\mathbf{N}} : 1 \leq |\vec{x}| \leq 2\}$. The analysis extends to any domain $\{\vec{x} \in \mathbf{R}^{\mathbf{N}} : 0 < \varepsilon \leq |\vec{x}| \leq L < \infty\}$. For system (1.1), the initial conditions are prescribed as

$$\rho_i(\vec{x}, 0) = \rho_{i0}(\vec{x}) \geq 0, \quad \vec{m}_i(\vec{x}, 0) = \vec{m}_{i0}(\vec{x}), \quad i = 1, 2, \tag{1.2}$$

and the boundary conditions are

$$\vec{m}_i(\vec{x}, t)\Big|_{\partial\Omega} = \vec{0}, \nabla\phi(\vec{x}, t)\Big|_{\partial\Omega} = \vec{0}, \text{ for } i = 1, 2, \ t \geq 0. \tag{1.3}$$

Moreover, the compatibility condition

$$\vec{m}_{i0}\Big|_{\partial\Omega} = \vec{0}, \quad i = 1, 2 \tag{1.4}$$

satisfies.

The model (1.1) consists of the conservation laws for the particle densities and the current densities, coupled to the Poisson equation for the electrostatic potential. It is well known that, due to the nonlinear hyperbolicity, the corresponding solutions of (1.1) with large initial data will develop singularities in finite time [27]. And then system (1.1) need to be considered in a weak sense. As far as 1-D spacial domain and weak solutions are concerned, Natalini [25] first proved the existence of entropy weak solutions and showed the convergence of the entropy solutions to the solutions of the corresponding classical **drift-diffusion** equations when the relaxation time goes to zero. This result was then extended to the bounded domain case by Hsiao and Zhang [7]. When the doping profile is completely flat (i.e. $D(x) = 0$) Huang and Li [8] showed that any bounded entropy solutions to the Cauchy problem of the bipolar hydrodynamic model converge to the **self-similar solutions** of the corresponding porous media equations as $t \to \infty$. As far as the authors know, there is **no result** on the large time behavior of weak solutions to problem (1.1)–(1.3) when the spacial dimension is more than one and the doping profile $D(x) \neq 0$. This paper is the first one to consider this problem. In the procedure, two essential points must be considered:

1. What should be the asymptotic profiles for the original solutions when the doping profile is not flat?
2. How to prove the weak solutions with less regularity convergence to the corresponding asymptotic profiles?

Inspired by the work of [4,10,24], in this paper, we show the L^∞ weak solutions with spherical symmetry converge to the **stationary solutions** in L^2 norm and derive the exponential decay rate. Compared with the case of smooth solutions, there are no smallness and regularity conditions assumed and the doping profile are permitted to be of large variation.