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On the energy estimates of the wave equation with time dependent propagation speed asymptotically monotone functions

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ABSTRACT

We consider the energy estimates for the wave equation with time dependent propagation speed. It is known that the asymptotic behavior of the energy is determined by the interactions of the properties of the propagation speed: smoothness, oscillation and the difference from an auxiliary function. The main purpose of the article is to show that if the propagation speed behaves asymptotically as a monotone decreasing function, then we can extend the preceding results to allow faster oscillating coefficients. Moreover, we prove that the regularity of the initial data in the Gevrey class can essentially contribute for the energy estimate.

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1. Introduction

Let us consider the following Cauchy problem of the wave equation with time dependent propagation speed:

$$\begin{cases} \left(\partial_t^2 - a(t)^2\Delta\right) u = 0, & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n, \\ (u(0,x), (\partial_t u)(0,x)) = (u_0(x), u_1(x)), & x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where $\mathbb{R}_+ = [0, \infty)$, $a(t) \in C^m(\mathbb{R}_+)$ with $m \geq 2$ satisfy $a(t) > 0$ and $\sup_t\{a(t)\} < \infty$. Here the total energy of (1.1) at t is defined by

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$$E(t) = E(t; u_0, u_1) := \frac{1}{2}\left(a(t)^2\|\nabla u(t,\cdot)\|^2 + \|\partial_t u(t,\cdot)\|^2\right),$$

where $\|\cdot\|$ denotes the usual L^2 norm in $\mathbb{R}^n$. If the propagation speed $a(t)$ is a constant, then the energy conservation $E(t) \equiv E(0)$ is valid. On the other hand, the energy conservation does not hold in general for variable propagation speed. However, the following equivalence between $E(t)$ and $E(0)$, which is called the *generalized energy conservation*:

$$C^{-1}E(0) \le E(t) \le CE(0) \tag{GEC}$$

with a constant $C > 1$, can be expected even though the propagation speed is not a constant. For instance, if $\inf_t\{a(t)\} > 0$ and $a'(t) \in L^1(\mathbb{R}_+)$, then (GEC) is trivial by the classical energy method, which is derived by the inequality $E'(t) \le C|a'(t)|E(t)$ and Gronwall's inequality. On the other hand, the classical energy method is useless for the proof of (GEC) if $a'(t) \notin L^1(\mathbb{R}_+)$. Actually, the L^1 property of $a'(t)$ is not enough to decide whether (GEC) is valid or not because both cases are possible if $a'(t) \notin L^1(\mathbb{R}_+)$; thus we introduce additional properties of $a(t)$. Let us suppose that

$$\inf_t\{a(t)\} > 0. \tag{1.2}$$

For $\alpha \in [0,1]$ and $\beta \in \mathbb{R}$ we introduce the following conditions:

$$\int_0^t |a(s) - a_\infty|\, ds \le C_0(1+t)^\alpha \tag{1.3}$$

and

$$\left|a^{(k)}(t)\right| \le C_k(1+t)^{-\beta k} \quad (k = 1, \ldots, m) \tag{1.4}$$

for some constants a_∞, $C_1, \ldots, C_m$; we shall denote universal positive constants by C and C_k with $k = 0, 1, \ldots$ without making any confusion. Here we remark the following:

- If $\alpha = 1$, then (1.3) is trivial for any constant a_∞. On the other hand, a_∞ is uniquely determined if (1.3) holds for $\alpha < 1$.
- If (1.4) holds for $\beta > 1$, then (GEC) is trivial because $a'(t) \in L^1(\mathbb{R}_+)$.
- (1.3) and (1.4) impose more restricted conditions as α smaller and β larger, respectively.

(1.3) is called the *stabilization property*, which describes an order of difference between the variable and constant propagation speeds, and (1.4) describes an order of oscillation and the smoothness of $a(t)$. Under the assumptions above, we have the following result:

Theorem 1.1. *(See [7].) Suppose that (1.2), (1.3) and (1.4) are valid. If α, β and m satisfy*

$$\beta \ge \beta_m := \alpha + \frac{1-\alpha}{m}, \tag{1.5}$$

then (GEC) is established. If $\beta < \alpha$, then (GEC) does not hold in general. (See Table 1.)

By Theorem 1.1 we see that (GEC) is determined by the interaction of the stabilization, the oscillation and the smoothness properties of $a(t)$. For instance, β can be taken smaller as α smaller and m larger. That is, faster oscillation can be admitted for (GEC) if $a(t)$ is smoother and strongly stabilized by a constant a_∞ in the sense of (1.3).

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