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Integrability and boundedness of solutions to some anisotropic problems

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ABSTRACT

In this article, we give a series of results on the improved integrability and boundedness of solutions to several anisotropic problems. In particular, we consider the nonhomogeneous Dirichlet problem for anisotropic elliptic second-order equations. Moreover, we study anisotropic variational inequalities with unilateral and bilateral obstacles as well as gradient constraints. Finally, variational problems for integral functionals with anisotropic integrands are considered.

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1. Introduction and exposition of known results

In the last decades, increasing attention has been given to the investigation of existence and properties of solutions to various problems for integral functionals with anisotropic integrands and for elliptic equations and variational inequalities with anisotropic coefficients (see for instance [1–11,13,15]). On the whole, the study of these problems requires more delicate procedures as compared with those used in the isotropic case. For the mentioned and close problems, one of the natural questions is the investigation of how the integrability of their solutions is changed in dependence on the improvement of integrability of their data (right-hand sides of equations, boundary data, etc.). Such an investigation was undertaken in the recent works of Leonetti and Siepe [10,11], Innamorati and Leonetti [6], and Gao et al. [4].

Let us discuss the main theorems proved in [4,6,10,11], since this will be connected with the principal results of the present article.

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In [10], Leonetti and Siepe considered the Dirichlet problem

$$\sum_{i=1}^{n} D_i a_i(x, \nabla u) = 0 \quad \text{in } \Omega, \qquad u = g \quad \text{on } \partial\Omega, \tag{1.1}$$

where Ω is a bounded open set of $\mathbb{R}^n$, $n \geqslant 2$, and the coefficients a_i satisfy the growth condition: there exist $p_1, \dots, p_n \in (1, +\infty)$ and $c' > 0$ such that for every $i \in \{1, \dots, n\}$, for almost every $x \in \Omega$, and for every $\xi \in \mathbb{R}^n$,

$$|a_i(x, \xi)| \leqslant c'(1 + |\xi_i|)^{p_i - 1}, \tag{1.2}$$

and the strong monotonicity condition: there exists $c'' > 0$ such that for almost every $x \in \Omega$ and for every $\xi, \xi' \in \mathbb{R}^n$,

$$\sum_{i=1}^{n} [a_i(x, \xi) - a_i(x, \xi')](\xi_i - \xi'_i) \geqslant c'' \sum_{i=1}^{n} |\xi_i - \xi'_i|^{p_i}. \tag{1.3}$$

In connection with the given problem, the anisotropic Sobolev space $W^{1,(p_i)}(\Omega)$ is defined as the set of all functions $v \in W^{1,1}(\Omega)$ such that $D_i v \in L^{p_i}(\Omega)$, $i = 1, \dots, n$. Analogously, the space $W_0^{1,(p_i)}(\Omega)$ is defined as the set of all functions $v \in W_0^{1,1}(\Omega)$ such that $D_i v \in L^{p_i}(\Omega)$, $i = 1, \dots, n$. Denoting by p_m the maximum of the numbers p_i and by $\bar{p}$ their harmonic mean, assuming that $\bar{p} < n$, setting $\bar{p}^* = n\bar{p}/(n - \bar{p})$, and supposing that $g \in W^{1,(q_i)}(\Omega)$ with $q_i \in (p_i, +\infty)$, $i = 1, \dots, n$, for the solution $u \in g + W_0^{1,(p_i)}(\Omega)$ of problem (1.1), Leonetti and Siepe established the following result (see [10, Theorem 2.1]): $u \in g + L^t_{weak}(\Omega)$, where

$$t = \frac{\bar{p}\,\bar{p}^*}{\bar{p} - b\bar{p}^*}, \tag{1.4}$$

and b is any positive number satisfying the inequalities

$$b < \frac{\bar{p}}{\bar{p}^*} \tag{1.5}$$

and

$$b \leqslant \frac{p_m}{p_m - 1} \min_{1 \leqslant i \leqslant n} \left(1 - \frac{p_i}{q_i}\right)\left(1 - \frac{1}{p_i}\right). \tag{1.6}$$

The result obtained means that higher integrability of the derivatives $D_i g$ of the boundary datum g implies that the function $u - g$ has improved integrability as compared with that given directly by the inclusion $u - g \in W_0^{1,(p_i)}(\Omega)$ and the imbedding of $W_0^{1,(p_i)}(\Omega)$ into $L^{\bar{p}^*}(\Omega)$.

Later on in [6], Innamorati and Leonetti considered problem (1.1) under condition (1.2) and the less restrictive than (1.3) coercivity condition: there exists $c''' > 0$ such that for almost every $x \in \Omega$ and for every $\xi \in \mathbb{R}^n$,

$$\sum_{i=1}^{n} a_i(x, \xi)\xi_i \geqslant c''' \sum_{i=1}^{n} |\xi_i|^{p_i}. \tag{1.7}$$

Based on the same assumptions on the exponents p_i and the function g as in [10], they established (see [6, Theorem 1.1]) that if $u \in g + W_0^{1,(p_i)}(\Omega)$ is an arbitrary solution of problem (1.1), then $u \in g + L^t_{weak}(\Omega)$, where t is defined by (1.4), and b is any positive number satisfying inequality (1.5) and the inequality

$$b \leqslant \min_{1 \leqslant i \leqslant n} \left(\frac{1}{p_i} - \frac{1}{q_i}\right). \tag{1.8}$$

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