J. Math. Anal. Appl. 432 (2015) 945–953

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Explosive instabilities for a generalized second grade fluid

M. Ciarletta [a], B. Straughan [b], V. Tibullo [c,*]

[a] *Dipartimento di Ingegneria Industriale, Università di Salerno, Italy*
[b] *Department of Mathematical Sciences, University of Durham, United Kingdom*
[c] *Dipartimento di Ingegneria dell'Informazione, Ingegneria Elettrica e Matematica Applicata, Università di Salerno, Italy*

A R T I C L E   I N F O

Article history:
Received 3 February 2015
Available online 15 July 2015
Submitted by D. Wang

Keywords:
Generalized second grade fluid
Finite time blow up
Shear thickening
Global nonexistence

A B S T R A C T

We consider a general model for a non-Newtonian fluid of second grade which is capable of describing shear thinning and shear thickening effects. The model incorporates parameters m and n which are varied to account for which effect, shear thinning or thickening, is desired in the tangential and normal shear rate directions, although here we restrict attention to the shear thickening case. By constructing a generalized energy-like equation and manipulating this we show that the solution may exhibit finite-time blow up if the initial data are regular enough for the solution to exist for a sufficiently long time. The nature of the blow up, or nonexistence, behavior depends critically on the normal stress coefficients α_1 and α_2, and on the parameters m and n.

1. Introduction

There has been increasing interest in models for viscoelastic fluid flows and general viscoelastic effects from a variety of standpoints, see e.g. Amendola and Fabrizio [1], Amendola et al. [2,3], Fabrizio and Lazzari [7], Franchi et al. [10,11], Jordan and Puri [13], Jordan and Saccomandi [14], Jordan and Keiffer [12], Man [16], Man and Sun [18], Man and Massoudi [17], Massoudi and Vaidya [21], Massoudi et al. [22], Massoudi and Phuoc [19,20], Phuoc and Massoudi [23], Phuoc et al. [24]. Of particular interest to the present article is the work on a model for a generalized fluid of second grade which is capable of capturing the effects of shear thinning and shear thickening, see Man and Sun [18], Man [16], Man and Massoudi [17], Massoudi et al. [22], Massoudi and Phuoc [19,20], Massoudi and Vaidya [21], Phuoc and Massoudi [23], Phuoc et al. [24]. Especially important in this context is the work of Massoudi and Phuoc [20], Phuoc and Massoudi [23], and Phuoc et al. [24] in employing a generalized second grade fluid model to capture correct physics for mass transfer in nanofluids. Since nanofluids are suspensions of fine particles such as Aluminum Oxide, Copper,

* Corresponding author.
E-mail addresses: mciarletta@unisa.it (M. Ciarletta), brian.straughan@durham.ac.uk (B. Straughan), vtibullo@unisa.it (V. Tibullo).

http://dx.doi.org/10.1016/j.jmaa.2015.07.015

or Copper oxide, a non-Newtonian model is to be expected. Given the intense recent interest in nanofluids we here study properties of the solution to a model for a generalized second grade fluid.

Let v_i be the velocity field and $L_{ij} = v_{i,j}$ be the velocity gradients, where $v_{i,j} = \partial v_i/\partial x_j$ and standard indicial notation is used throughout. When we refer to the velocity vector we write this as $\mathbf{v}$. Then the classical model for a fluid of second grade consists of the momentum and continuity equations

$$\rho \dot{v}_i = \rho f_i + T_{ji,j} \tag{1}$$

and

$$v_{i,i} = 0, \tag{2}$$

see e.g. Dunn and Fosdick [6]. Here $\dot{v}_i = \partial v_i/\partial t + v_j \partial v_i/\partial x_j$ denotes the material derivative, f_i is the body force, and $T_{ij} = T_{ji}$ is the Cauchy stress tensor.

Let $\mathbf{A}_1$ and $\mathbf{A}_2$ denote the first two Rivlin–Ericksen tensors, so that

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \qquad \mathbf{A}_2 = \dot{\mathbf{A}}_1 + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1, \tag{3}$$

then the constitutive equation for a classical fluid of second grade is, cf. Dunn and Fosdick [6],

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2. \tag{4}$$

These writers deduce that the viscosity $\mu \geq 0$, and the normal stress coefficients α_1 and α_2 are subject to the restrictions

$$\alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0. \tag{5}$$

However, conditions (5) have been disputed in the literature, see e.g. Fosdick and Rajagopal [8], and here we consider the possibility that $\alpha_1 > 0$, or $\alpha_1 < 0$, and also $\alpha_1 + \alpha_2 \neq 0$.

Man and Sun [18] and later Man [16], Man and Massoudi [17], Massoudi and Phuoc [19,20], Massoudi and Vaidya [21], Phuoc and Massoudi [23], Phuoc et al. [24], proposed a modification to the constitutive equation (4) in order to encompass shear thinning and thickening effects. They suggest using

$$\mathbf{T} = -p\mathbf{I} + \mu\Pi^{\frac{m}{2}}\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{6}$$

or

$$\mathbf{T} = -p\mathbf{I} + \Pi^{\frac{m}{2}}\left(\mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2\right) \tag{7}$$

where $\Pi = \operatorname{tr}\mathbf{A}_1^2$, and m is a real number. When $m > 0$ equations (6) and (7) are employed to describe shear thickening whereas when $m < 0$ shear thinning is possible. In this work we restrict attention to the shear thickening case.

To describe thermal convection with a generalized second grade fluid Franchi and Straughan [9] employed the equation

$$\mathbf{T} = -p\mathbf{I} + \mu(T)[1 + \hat{\gamma}(\operatorname{tr}\mathbf{A}_1^2)^{\frac{m}{2}}]\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{8}$$

where T is temperature. Effectively the same equation for constant temperature is proposed by Massoudi and Vaidya [21], eq. (36). This extension to equation (6) is important in that it allows one to recover a