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Nonlocal elliptic equations involving measures

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ABSTRACT

In this article, we study the existence of solution for the problem $(-\Delta)^\alpha u = \lambda f(u) + \nu$ in Ω, $u \equiv 0$ in $\mathbb{R}^N \backslash \Omega$, where $\lambda > 0$ is a parameter, $\alpha \in (0, 1)$ and ν is a Radon measure. A weak solution is obtained by using Schauder's fixed point theorem. In the case where ν is Dirac measure, the symmetry of the solution is obtained by using the moving plane method.

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1. Introduction

In recent years, fractional (nonlocal) Laplacian operator has been extensively studied by many authors [7,8,6,2,10,16,17,15]. It is well known that the corresponding Fokker–Planck equation to a stochastic differential equation with Brownian motion is the traditional diffusion equation. When the Brownian motion is replaced by an α-stable Lévy motion (a non-Gaussian process) L_t^α, $\alpha \in (0, 2)$, the Fokker–Planck equation becomes a nonlocal partial differential equation [1] with a fractional Laplacian operator $(-\Delta)^{\frac{\alpha}{2}}$. There are many physical motivations to consider the fractional Laplacian operator, which appears in many models in non-Newtonian fluids, in models of viscoelasticity such as Kelvin–Voigt models, various heat transfer processes in fractal and disordered media and models of fluid flow and acoustic propagation in porous media [3,13,14]. Interestingly, it has also been applied to pricing derivative securities in financial market, see [3] for details.

Recently, Chen and Véron [8] considered the following problem:

$$\begin{cases} (-\Delta)^\alpha u + g(u) = \nu, \quad t > 0, x \in \Omega, \\ u|_{\Omega^c} = 0, \end{cases} \tag{1.1}$$

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where $\Omega \subset \mathbb{R}^N$ is an open bounded C^2 domain, $\alpha \in (0,1)$, ν is a Radon measure such that $\int_\Omega \delta^\beta d|\nu| < \infty$ for some $\beta \in [0,\alpha]$ and $\delta(x) = \mathrm{dist}(x, \Omega^c)$. The nonlocal Laplacian $(-\Delta)^\alpha$ is defined by

$$(-\Delta)^\alpha u(x) = \lim_{\varepsilon \downarrow 0} (-\Delta)^\alpha_\varepsilon u(x),$$

where for $\varepsilon > 0$

$$(-\Delta)^\alpha_\varepsilon u(x) = c_{n,\alpha} \int_{\mathbb{R}^N} \frac{u(x) - u(y)}{|x-y|^{n+2\alpha}} \chi_\varepsilon(|x-y|) dy,$$

and

$$\chi_\varepsilon(x) = \begin{cases} 0, & \text{if } x \in [0,\varepsilon], \\ 1, & \text{if } x > \varepsilon. \end{cases}$$

They proved that (1.1) admits a unique weak solution u under the condition that $g : \mathbb{R} \to \mathbb{R}$ is a continuous, non-decreasing function, satisfying

$$g(r)r \geq 0, \quad \forall r \in \mathbb{R} \ \text{ and } \int_1^\infty (g(s) - g(-s)) s^{-1-k_{\alpha,\beta}} ds < \infty,$$

where

$$k_{\alpha,\beta} = \begin{cases} \dfrac{N}{N-2\alpha}, & \text{if } \beta \in [0, \dfrac{N-2\alpha}{N}\alpha], \\ \dfrac{N+\alpha}{N-2\alpha+\beta}, & \text{if } \beta \in (\dfrac{N-2\alpha}{N}\alpha, \alpha]. \end{cases} \tag{1.2}$$

In their another paper [7], they obtained the existence of weak solution to (1.1), where $g(u)$ was replaced by $\varepsilon g(|\nabla u|)$, $\varepsilon = \pm 1$. When the measure ν is just a bounded function $g(x)$, the existence of solutions to (1.1) has been studied in [18] via variational methods.

Ros-Oton and Serra [16] studied the extremal solution for the following problem:

$$\begin{cases} (-\Delta)^\alpha u = \lambda f(u), & \text{in } \Omega, \\ u|_{\Omega^c} = 0, \end{cases} \tag{1.3}$$

where $\lambda > 0$ is a parameter, $\alpha \in (0,1)$ and $f : [0,\infty) \to \mathbb{R}$ satisfies

$$f \in C^1, \quad \text{non-decreasing}, \quad f(0) > 0, \text{ and } \lim_{t \to +\infty} \frac{f(t)}{t} = +\infty. \tag{1.4}$$

Under the above assumptions, they proved that there exists $\lambda^* \in (0,\infty)$ such that

(i) If $0 < \lambda < \lambda^*$, problem (1.3) admits a minimal classical solution u_λ;
(ii) The family of functions $\{u_\lambda : 0 < \lambda < \lambda^*\}$ is increasing in λ, and its pointwise limit $u^* = \lim_{\lambda \uparrow \lambda^*}$ is a weak solution of (1.3) with $\lambda = \lambda^*$;
(iii) For $\lambda > \lambda^*$, problem (1.3) admits no classical solution.

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