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Removable singularities for the Von Karman equations

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A B S T R A C T

A theorem for removable singularities of the fourth order system of equations in the plane called the Von Karman equations is established. This result is best possible in several different ways. The key idea is to apply the *Calderon and Zygmund* singular integral theory in conjunction with several lemmas to a nontrivial bootstrap argument.

1. Introduction

We will operate in real two-dimensional Euclidean space, $\mathbf{R}^2$, and use the following notation:

$$x = (x_1, x_2), \qquad y = (y_1, y_2)$$
$$\alpha x + \beta y = (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2)$$
$$x \cdot y = x_1 y_1 + x_2 y_2, \qquad |x| = (x \cdot x)^{\frac{1}{2}}.$$

Also, $D(x, \rho)$ will designate the open disk centered at x with radius ρ. $D(x, \rho)\setminus\{x\}$ will designate the corresponding disk punctured at x. For partial derivatives, we will sometimes use the notation

$$\frac{\partial u}{\partial x_1} = u_1, \ \frac{\partial^2 u}{\partial x_1^2} = u_{11}, \ \frac{\partial^2 u}{\partial x_1 \partial x_2} = u_{12}.$$

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For $\Omega \subset \mathbf{R}^2$ a bounded open set with $\delta(\Omega)$ designating the diameter of Ω and with $F \in C(\Omega)$, we designate the modulus of continuity of F by $\omega(t, F, \Omega)$ where

$$\omega(t, F, \Omega) = \sup\{|F(x) - F(y)| : x, y \in \Omega,\ |x - y| \leq t\}.$$

With $\Omega \subset \mathbf{R}^2$ an open set, we say $F \in C^\omega(\Omega)$ provided $F \in C(\Omega)$ and

$$\int_0^{\delta(\Omega_1)} \omega(t, F, \Omega_1)/t\, dt < \infty$$

for every open set Ω_1 whose closure is a compact subset of Ω. If $F \in C^\omega(\Omega)$, we say F satisfies a Dini condition in Ω.

We say $F \in C^{2+\omega}(\Omega)$ if $F \in C^2(\Omega)$ and $F_{11}, F_{12}, F_{22} \in C^\omega(\Omega)$.

For the Von Karman equations, we will take the following fourth order system: in the open set Ω,

$$\begin{cases} \Delta^2 f = -[w, w] \\ \Delta^2 w = [F, w] + [f, w] \end{cases} \quad \text{in } \Omega, \tag{1.1}$$

where $[w, f] = w_{11}f_{22} + w_{22}f_{11} - 2w_{12}f_{12}$. Here, F is a given function in $C^{2+\omega}(\Omega)$. Also, Δ^2 stands for the iterated Laplace operator.

In the literature f is called a stress function and w is called a deflection.

It is our intention to establish the following result concerning removable singularities for the Von Karman system (1.1).

Theorem. *With $\Omega = D(0, \rho_0)$, $\rho_0 > 0$, assume $f, w \in C^4(\Omega \backslash \{0\})$ and $F \in C^{2+\omega}(\Omega)$. Assume furthermore that f, w satisfy the Von Karman system (1.1) in $\Omega \backslash \{0\}$. Also, suppose that*

(i) $f, \Delta f = o\left(\log \frac{1}{r}\right)$ *as* $r \to 0$,
(ii) $w, \Delta w = o\left(\log \frac{1}{r}\right)$ *as* $r \to 0$.

Then both f and w can be defined at 0 so that $f, w \in C^4(\Omega)$ and f, w satisfy the Von Karman system (1.1) in Ω.

It turns out that in several ways the above theorem is a best possible result. If (i) $f = o\left(\log \frac{1}{r}\right)$ is replaced with $f = O\left(\log \frac{1}{r}\right)$ and Δf remains $o\left(\log \frac{1}{r}\right)$ and (ii) remains the same, then a counter-example to the theorem holds. Likewise, there is a counter-example if $\Delta f = O\left(\log \frac{1}{r}\right)$ and f remains $o\left(\log \frac{1}{r}\right)$ and (ii) remains the same.

The first of these counter-examples is obtained by choosing

$$f(x) = \log \frac{1}{|x|} \text{ and } w = 0.$$

The second is obtained by choosing

$$f(x) = |x|^2 \log \frac{1}{|x|} \text{ and } w = 0.$$

Also a counter-example exists if in (ii) $\Delta w = O\left(\log \frac{1}{r}\right)$ and w remains $o\left(\log \frac{1}{r}\right)$ and (i) remains the same. This counter-example requires the technique used by Berger in his paper, [2]. We will present this result in a sequel to this current manuscript.