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On nonlinear wave equations of Carrier type

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ABSTRACT

This paper is concerned with the initial-boundary value problem of the n-dimensional Carrier equation with an internal damping. This damping is a fractional power of the velocity of the points of the material. The Faedo–Galerkin method, Tartar approach and compactness arguments provide the global existence of solutions of the above problem with restriction on the size of the initial data. The decay of solutions is obtained by the perturbation method.

1. Introduction

We consider the Carrier operator [3] and [4], defined by

$$u \mapsto \frac{\partial^2 u}{\partial t^2} - M\left(\int_\Omega u^2\,dx\right)\Delta u. \tag{1.1}$$

Note that $u = u(x,t)$ represents a real function defined for $(x,t) \in Q = \Omega \times (0,\infty)$, Ω a non-empty open bounded set of $\mathbb{R}^n$ with smooth boundary Γ. Thus, Q is an open cylinder of $\mathbb{R}^{n+1}$ with lateral boundary $\Sigma = \Gamma \times (0,\infty)$.

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In the present article we investigate the following nonlinear mixed problem of Carrier type:

$$\left|\begin{array}{l} \dfrac{\partial^2 u}{\partial t^2} - M\left(\displaystyle\int_\Omega u^2\,dx\right)\Delta u + \delta\left|\dfrac{\partial u}{\partial t}\right|^\rho \dfrac{\partial u}{\partial t} = 0 \quad \text{in } Q; \\ u = 0 \quad \text{on } \Sigma; \\ u(x,0) = u^0(x), \quad \dfrac{\partial u}{\partial t}(x,t) = u^1(x) \quad \text{for } x \in \Omega, \end{array}\right. \tag{$*$}$$

with $0 \leq \rho < 1$ and $\delta > 0$ a parameter.

By $\dfrac{\partial}{\partial t}$ we represent the symbol of partial derivative with respect to $t > 0$, by Δ the Laplace operator, that is, $\Delta u = \sum_{i=1}^{n} \dfrac{\partial^2 u}{\partial t^2}$. All derivatives, that we consider, are in the sense of the theory of distributions, see [9] and [12].

The above mixed problem (∗) was investigated from mathematical point of view in [1,11,10] among others. We fix our attention on the results contained in [1,10] with the nonlinearity $\delta\left|\dfrac{\partial u}{\partial t}\right|\left|\dfrac{\partial u}{\partial t}\right|^\rho$ with the restriction $\rho > 1$. For a motivation of this type of nonlinearity see [8], p. 38, when $M(\lambda) = 1$. The authors in [11] analyzed also the case $\rho = 1$. The main difficulty in [1,11] is that the proofs depend on the Young–Hölder inequality which is true only for $\rho > 1$. For $\rho = 1$ the authors of [11] assume more restriction. They did not treat the case $0 \leq \rho < 1$ which is our objective in this paper.

We overcome the difficulty motivated by application of Young–Hölder's inequality, employing an argument by contradiction, which works well if we restrict the initial data u^0, u^1 to be inside a ball. This argument can be found in [12,15]. For similar questions, see [1,5].

The plan of the paper is as follows. In Section 2, we introduce the notations, hypotheses in order to state the theorem on the existence of solutions. In Section 3, we prove the above theorem. In Section 4, we investigate the decay of solutions and in Section 5, we give some examples of functions $M(\lambda)$ satisfying all the required conditions.

2. Notations, hypotheses and existence of solutions

We represent by $L^2(\Omega)$ the Lebesgue space of real functions u which are square integrable in Ω, with scalar product and norm defined by:

$$(u,v) = \int_\Omega uv\,dx \quad \text{and} \quad |u|^2 = \int_\Omega |u|^2\,dx.$$

By $H^m(\Omega)$ we denote the Sobolev space of order $m \in \mathbb{N}$. By $H_0^1(\Omega)$ we represent the distributions of $H^1(\Omega)$ which have trace zero on Γ, for example see [13]. The scalar product and norm in $H_0^1(\Omega)$ are given by:

$$((u,v)) = \int_\Omega \nabla u \cdot \nabla v\,dx \quad \text{and} \quad ||u||^2 = \int_\Omega |\nabla u|^2\,dx.$$

We also consider the Banach space $L^p(\Omega)$, $p \in \mathbb{R}$, $p \geq 1$; in particular we consider $p = \rho + 2$, $0 \leq \rho < 1$. We have the know results:

$$|v| \leq a_0 ||v||_{L^{\rho+2}(\Omega)}, \quad \forall v \in L^{\rho+2}(\Omega)$$