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Global existence of radial solutions for general semilinear hyperbolic systems in 3D

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ABSTRACT

We study the well-posedness of radial solutions for general nonlinear hyperbolic systems in three dimensions. We give a proof of the global existence of radial solutions for general semilinear hyperbolic systems in 3D under null condition, with small scaling invariant $\dot{W}^{2,1}(\mathbb{R}^3)$ data. We obtain a bilinear estimate that is effective for hyperbolic systems which do not have any time decay. It allows us to achieve the boundedness of the weighted BV norm of the radial solutions.
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1. Introduction and main result

We consider the following general first-order semilinear hyperbolic system

$$\frac{\partial u}{\partial t} + \sum_{i=1}^{3} A_i \frac{\partial u}{\partial x^i} = Q(u, u), \tag{1.1}$$

where $u = (\rho_1, \cdots, \rho_l, v_1, \cdots, v_m)^T$ is the unknown function of (t, x^1, x^2, x^3), $\rho_1, \cdots, \rho_l$ are scalar unknown functions valued in $\mathbb{R}$, and $v_1, \cdots, v_m$ are vector unknown functions valued in $\mathbb{R}^3$. A_1, A_2, A_3 are constant matrixes and $Q(u, u) = (Q^\rho(u, u), Q^v(u, u))^T$, $Q^\rho = (Q_1^\rho, \cdots, Q_l^\rho)$, $Q^v = (\mathbf{Q}_1^v, \cdots, \mathbf{Q}_m^v)^T$, where each $Q_k^\rho, \mathbf{Q}_p^v = (Q_p^{v1}, Q_p^{v2}, Q_p^{v3})$ is quadratic in u.

System (1.1) is equipped with the following initial data

$$t = 0: \qquad u = u_0(x), \qquad x \in \mathbb{R}^3. \tag{1.2}$$

To describe the space of the initial data, we introduce

$$D = \{\partial_1, \partial_2, \partial_3\},$$

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and

$$\dot{W}^{2,1}(\mathbb{R}^3) = \{u : \|D^2 u\|_{L^1(\mathbb{R}^3)} < \infty\}.$$

Let us take a look at the invariant group of the system (1.1). Suppose that $u(t,x)$ satisfies system (1.1), so that obviously $u^\lambda(t,x)$ also satisfies system (1.1), which is defined by

$$u^\lambda(t,x) = \lambda u(\lambda t, \lambda x),$$
$$u_0^\lambda = \lambda u_0(\lambda x),$$

for all $\lambda > 0$. Clearly, the space $\dot{W}^{2,1}(\mathbb{R}^3)$ for this problem is dimensionless that is scaling invariant

$$\|u_0\|_{\dot{W}^{2,1}(\mathbb{R}^3)} = \|u_0^\lambda\|_{\dot{W}^{2,1}(\mathbb{R}^3)}.$$

And we have

$$\frac{\partial u^\lambda}{\partial t} + \sum_{i=1}^{3} A_i \frac{\partial u^\lambda}{\partial x^i} = Q(u^\lambda, u^\lambda). \tag{1.3}$$

By differential (1.3) with respect to λ and then take $\lambda = 1$, we have

$$(\frac{\partial}{\partial t} + \sum_{i=1}^{3} A_i \frac{\partial}{\partial x^i})(L_0 + 2)u = (L_0 + 1)Q(u,u).$$

Here L_0 denotes the scaling operator that was introduced by Klainerman [10],

$$L_0 = t\partial_t + x^j \partial_j,$$

and we use Einstein's summation convention for repeated indices.

In order to state our result precisely, we need to make some assumptions. The first assumption is the following.

[H1]: Cauchy problem (1.1), (1.2) is rotationally invariant.

We know that many physical systems in three dimensions, such as compressible Euler equations, relativistic compressible Euler equations and compressible MHD equations etc., are invariant under rotation of coordinates. The goal in this paper is to consider the global existence of radial solutions to Cauchy problem (1.1), (1.2), which is invariant under rotation of coordinates.

Let's consider the corresponding hyperbolic system in one dimension

$$\frac{\partial u}{\partial t} + A_1 \frac{\partial u}{\partial x^1} = Q(u,u), \tag{1.4}$$

where $u, A_1, Q(u,u)$ are defined as before in (1.1).

We recall the following concept.

Definition 1.1. The system (1.4) satisfies *null condition* if any small plane wave solution $u = u(t + \xi x^1)$ (ξ is a constant in $\mathbb{R}$) to the linearized system

$$\frac{\partial u}{\partial t} + A_1 \frac{\partial u}{\partial x^1} = 0$$

is always a solution to the original system (1.4).

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