



Analysis of a general stochastic non-autonomous logistic model with delays and Lévy jumps



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ARTICLE INFO

Article history:

Received 20 August 2014
Available online 21 July 2015
Submitted by J. Shi

Keywords:

Persistence
Extinction
Lévy noise
Delays
Lyapunov functions

ABSTRACT

In this paper, a general stochastic non-autonomous logistic model with delays and Lévy jumps is proposed and studied. Sufficient criteria for extinction, non-persistence in the mean and weak persistence of the model are established. The threshold between weak persistence and extinction is also obtained. Finally, numerical simulations are introduced to illustrate the theoretical analysis results. Our results illustrate that both persistence and extinction have close relationships with time-dependent delay and Lévy noise.

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1. Introduction

As far as we know, logistic model is one of the most important and classical models in mathematical biology. The deterministic non-autonomous logistic model with time-dependent delay and infinite delay can be expressed by the following ordinary differential equation:

$$\frac{dx(t)}{dt} = x(t) \left[r(t) - a(t)x(t) + b(t)x(t - \tau(t)) + c(t) \int_{-\infty}^0 x(t + \theta) d\mu(\theta) \right], \quad (1.1)$$

where $x(t)$ denotes the population size at time t , $\tau(t) \geq 0$ represents the time-dependent delay and $\mu(\cdot)$ is a probability measure on $(-\infty, 0]$, $r(t)$, $a(t)$, $b(t)$ and $c(t)$ are continuous bounded functions on $\bar{\mathcal{R}}_+ = [0, +\infty)$. For biological significance of each coefficient in system (1.1), we refer the reader to Gard [15,16]. A further and extensive property is considered in model (1.1) or systems similar to (1.1) containing persistence, extinction, global asymptotic stability and other dynamics. Here, we only refer to Gopalsamy [17,18], Kuang and Smith [27], Gakkhar and Singh [13], He and Gopalsamy [21], Lisena [30] and Kuang [26]. In particular, the books by Gopalsamy [18] and Kuang [26] are good references for the reader.

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However, in real life, population system is always affected by environmental noise (see e.g. [23,24,5,40,35]), which is an important component in an ecosystem. Using stochastic models can predict the future dynamics of the system accurately. Stochastic differential equation models play an important role in many kinds of branches of applied sciences including population dynamics, as they can provide some additional degree of realism compared to their corresponding deterministic counterpart. There are extensive literatures on functional SDEs (see e.g. [7,8,12]). May [43] revealed that due to environmental fluctuations, the birth rates, death rates, carrying capacity, competition coefficients and other parameters involved with the system exhibit random fluctuation to a greater or lesser extent. Hence, many scholars have introduced stochastic perturbations into deterministic models to reveal the effects of environmental noises on the population dynamics (see e.g. [14,28,36,37,41]).

Assume that the white noise affects $r(t)$, $a(t)$, $b(t)$ and $c(t)$, then by the central limit theorem, we can estimate $r(t)$, $a(t)$, $b(t)$ and $c(t)$ by an average value plus error terms which follow a normal distribution. In other words, we can substitute $r(t)$, $a(t)$, $b(t)$ and $c(t)$ with $r(t) + \sigma_1(t)\dot{w}_1(t)$, $-a(t) + \sigma_2(t)\dot{w}_2(t)$, $b(t) + \sigma_3(t)\dot{w}_3(t)$ and $c(t) + \sigma_4(t)\dot{w}_4(t)$, respectively. Here $\sigma_i(t)$ ($i = 1, 2, 3, 4$) represent the intensities of the white noise at time t , $\dot{w}_i(t)$ ($i = 1, 2, 3, 4$) are the white noises, that is to say, $w_i(t)$ ($i = 1, 2, 3, 4$) are Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathcal{P}$ -null sets). $\sigma_i(t)$ ($i = 1, 2, 3, 4$) are positive continuous bounded functions on $\bar{\mathcal{R}}_+$. Inspired by this, we obtain the following stochastic system:

$$\begin{aligned} dx(t) = & x(t) \left[r(t) - a(t)x(t) + b(t)x(t - \tau(t)) + c(t) \int_{-\infty}^0 x(t + \theta) d\mu(\theta) \right] dt \\ & + \sigma_1(t)x(t)dw_1(t) + \sigma_2(t)x^2(t)dw_2(t) + \sigma_3(t)x(t)x(t - \tau(t))dw_3(t) \\ & + \sigma_4(t)x(t) \int_{-\infty}^0 x(t + \theta) d\mu(\theta)dw_4(t). \end{aligned}$$

On the other hand, in the real world, population system may be a subject to sudden and severe environmental perturbations, such as earthquakes, epidemics, harvesting and so on (see e.g. [6,9,39,33,31]). These phenomena cannot be described better by Brownian motion. Introducing Lévy noise into the underlying population system may be a good way to describe these phenomena. There exists an interesting literature concerned with SDEs with jumps. We here only mention Bao et al. [6,9–11], Liu and Wang [39], Liu et al. [31,32], Applebaum et al. [1–3] and Situ [45]. By taking the effects of Lévy jumps into account, we get the following logistic model with Lévy jumps:

$$\begin{aligned} dx(t) = & x(t) \left[r(t) - a(t)x(t) + b(t)x(t - \tau(t)) + c(t) \int_{-\infty}^0 x(t + \theta) d\mu(\theta) \right] dt \\ & + \sigma_1(t)x(t)dw_1(t) + \sigma_2(t)x^2(t)dw_2(t) + \sigma_3(t)x(t)x(t - \tau(t))dw_3(t) \\ & + \sigma_4(t)x(t) \int_{-\infty}^0 x(t + \theta) d\mu(\theta)dw_4(t) + \int_{\mathbb{Y}} \gamma(t, y)x(t^-) \tilde{N}(dt, dy), \end{aligned} \quad (1.2)$$

where $x(t^-)$ denotes the left limit of $x(t)$, N represents a Poisson counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0, \infty)$ with $\lambda(\mathbb{Y}) < +\infty$ and we suppose that λ is a Lévy measure such that $\tilde{N}(dt, dy) = N(dt, dy) - \lambda(dy)dt$, $\mathbb{Y}$ is a measurable subset of $\mathcal{R}_+$.

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