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On a nonlinear thermoelastic system with nonlocal coefficients

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ABSTRACT

This paper deals with the global existence and uniqueness of solutions, and uniform stabilization of the energy of an initial–boundary value problem for a thermoelastic system with nonlinear terms of nonlocal kind. The asymptotic stabilization of the energy of system (1.1) is obtained without any dissipation mechanism acting in the displacement variable u of the membrane equation.

1. Introduction and main results

1.1. The model

Suppose that Ω is a bounded and open set in $\mathbb{R}^n$ having a smooth boundary Γ and lying at one side of Γ. Let $Q = \Omega \times (0, \infty)$ and $\Sigma = \Gamma \times (0, \infty)$ its lateral boundary.

The motivations significant to our article are contained in Chipot–Lovar [4] and Límaco et al. [17]. In what follows we comment in few words on these two works, in order to establish the problem to be studied here.

In Chipot–Lovar [4] was investigated a parabolic equation describing the average of the diffusion, for instance, of the temperature across its domain. More precisely, if $\theta = \theta(x, t)$ is the relative temperature, the equation

$$\theta' - a\left(l(\theta)\right)\Delta\theta = 0 \text{ in } Q \tag{i}$$

is an approximated model for such problems, where a is a bounded continuous function from $\mathbb{R}$ to $\mathbb{R}$, l is a continuous linear functional from $L^2(\Omega)$ to $\mathbb{R}$ that depends upon of the nonlocal diffusion coefficient

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$\int_\Omega \theta(t)\,dx$, $\Delta = \sum_{i=1}^{n} \partial^2_{x_i^2}$ is the Laplace operator and the prime symbol $'$ denotes the partial derivative with respect to t.

The nonlocal term of equation (i) is obtained upon application of a nonlocal Fourier-type law where the heat flux $\vec{q} = \vec{q}(x,t)$ depends on the temperature measurements made not locally, but on average across the conductor Ω. Thus, the material law can be stated as $\vec{q}(x,t) = -\kappa(t)\nabla\theta(x,t)$ where the function κ is given by $\kappa(t) = \kappa\big(\int_\Omega \theta(t)dx\big)$, i.e., the heat flux depends on the total thermal energy stored in Ω.

A straightforward generalization of the non-local diffusion coefficient of equation (i) is the operator $\mathcal{B}$ defined by

$$\mathcal{B}(t)\theta(\cdot,t) = -\sum_{i,j=1}^{n} B_{ij}\left(\theta(\cdot,t),t\right)\partial^2_{x_i\,x_j}\theta(\cdot,t) \quad \text{with} \quad B_{ij} : L^1(\Omega)\times[0,\infty[\,\to\mathbb{R}, \tag{ii}$$

which will be used throughout this work. This operator $\mathcal{B}$ was also considered in Fernandez-Cara et al. [10].

In Límaco et al. [17] was derived a model for small transverse vibrations of an elastic membrane assuming that the density of the material is not constant. More precisely, when $u = u(x,t)$ is the deformation of the membrane an approximate model for this physical phenomenon can be written for a dissipative equation of membrane type with variable coefficients like this

$$u'' - M\Big(\cdot,\cdot,\int_\Omega |\nabla u|^2_{\mathbb{R}}\,dx\Big)\Delta u + k(\cdot,\cdot)u' = 0 \ \text{ in } \ Q, \tag{iii}$$

where $M = M(x,t,\lambda) = M_1(x,t) + M_2(x,t,\lambda)$ and $k = k(x,t)$ are real functions defined on $\Omega\times[0,\infty)\times[0,\infty)$ and $\Omega\times[0,\infty)$, respectively.

If the momentum, ku', is neglected we then have the conservative membrane model

$$u'' - M\Big(\cdot,\cdot,\int_\Omega |\nabla u|^2_{\mathbb{R}}\,dx\Big)\Delta u = 0 \ \text{ in } \ Q. \tag{iv}$$

In some oscillatory phenomena, as described in equations (iii) and (iv), is physically important to consider the thermal effects caused by temperature across the material, and so arise coupled thermoelastic systems. In particular, a natural coupling of the equations (i) and (iv) is through of the operator $\mathbf{a}\cdot\nabla$ given by $\sum_{i=1}^{n} a_i\partial_{x_i}$, being $\mathbf{a} = (a_1,\ldots,a_n)$ a constant vector of $\mathbb{R}^n$.

The precedent considerations are the motivations for us investigate the following nonlinear coupled initial–boundary value system

$$\begin{aligned}
&u'' - M\left(\cdot,\cdot,|\nabla u|^2\right)\Delta u + (\mathbf{a}\cdot\nabla)\theta = 0 && \text{in} \quad Q,\\
&\theta' + \mathcal{B}\theta + (\mathbf{a}\cdot\nabla)u' = 0 && \text{in} \quad Q,\\
&u = \theta = 0 && \text{on} \quad \Sigma,\\
&u(x,0) = u_0(x), \ \ u'(x,0) = u_1(x), \ \ \theta(x,0) = \theta_0(x) && \text{in} \quad \Omega,
\end{aligned} \tag{1.1}$$

where all these derivatives in (1.1) are in the sense of the distributions of Schwartz [26] and $|\cdot|$ denotes the norm of the Lebesgue space $L^2(\Omega)$.

The main goals in this paper are to establish, under suitable conditions, the existence and uniqueness of strong global solutions for system (1.1), and that its energy decay uniformly to zero, as $t\to\infty$. The asymptotic stabilization of the energy of system (1.1) is obtained without any dissipation mechanism acting in the displacement variable u or its trace on the boundary. It is noteworthy that the assumptions herein are similar to those of Límaco et al. [17], wherein the membrane equation has internal dissipation mechanism, and this permits to establish the uniform stabilization of the energy of the system.