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Dynamics of food-chain models with density-dependent diffusion and ratio-dependent reaction function

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ABSTRACT

This paper is concerned with a 3-species and a 2-species food-chain reaction diffusion systems in a bounded domain where the diffusion coefficients may be density dependent and the reaction functions are ratio-dependent. These equations are quasilinear where the diffusion coefficients may be degenerate on the boundary of the domain. Three basic types of Dirichlet, Neumann and Robin boundary conditions are considered, and in each case some very simple conditions are obtained to ensure the dynamical behavior of the time-dependent solution in relation to some positive solutions or quasi-solutions of the steady-state problem, including the existence of these solutions. This dynamical behavior leads to the coexistence and global attractor of the food-chain systems. In the case of Neumann boundary condition sufficient conditions are given to ensure that the steady-state problem has a unique positive constant solution which is a global attractor of the time-dependent system.

1. Introduction

Understanding of spatial and temporal behavior of interacting species in ecological systems is a major concern in population dynamics. One important aspect of the problem is whether the various species in the system can persist at a coexistence state or even approach to a steady state. In the case of inhomogeneous environment the various population densities are governed by reaction diffusion equations, and the existence of a positive solution and its asymptotic behavior are of great concern in the ecological system. However, most of the reaction diffusion equations studied in the current literature are for semilinear parabolic and elliptic equations where the diffusion coefficients are density independent. In this paper, we investigate a coupled system of three reaction diffusion equations for some food-chain models where the diffusion coefficients are density-dependent and the reaction functions are ratio-dependent. Furthermore, the effect of diffusion may be degenerate when the boundary condition is of Dirichlet type. The system of equations under consideration is given in the form

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$$\begin{aligned}
&\partial u/\partial t - d_1(x)\,\Delta u^{m_1} = u\,[a_1 - b_1 u - c_1 v/(u+\mu_1 v)] \\
&\partial v/\partial t - d_2(x)\,\Delta v^{m_2} = v\,[-a_2 + b_2 u/(u+\mu_2 v) - c_2 w/(\mu_2' v + w)] \quad (t>0, \quad x\in\Omega) \\
&\partial w/\partial t - d_3(x)\,\Delta w^{m_3} = w\,[-a_3 + b_3 v/(\mu_3 v + w)] \\
&\alpha_1 \partial u/\partial\nu + \beta_1 u = 0, \quad \alpha_2 \partial v/\partial\nu + \beta_2 v = 0, \quad \alpha_3 \partial w/\partial\nu + \beta_3 w = 0 \quad (t>0, \quad x\in\partial\Omega) \\
&u(0,x) = \xi_1(x), \quad v(0,x) = \xi_2(x), \quad w(0,x) = \xi_3(x), \quad (x\in\Omega) \qquad (1.1)
\end{aligned}$$

where Ω is a bounded domain in $\mathbb{R}^p$ with boundary $\partial\Omega$ ($p = 1, 2, \ldots$), $\partial/\partial\nu$ denotes the outward normal derivative on $\partial\Omega$, and for each $i = 1, 2, 3$, $d_i(x) = \mathcal{C}^\alpha(\Omega)$ and is positive in $\bar{\Omega} \equiv \Omega \cup \partial\Omega$, and m_i, a_i, b_i, c_i, μ_i and μ_2' are positive constants with $m_i \geq 1$. (However, c_2 and μ_i are allowed to be zero.) The boundary coefficients α_i and β_i are nonnegative constants with $\alpha_i + \beta_i > 0$. This includes the three basic types of Dirichlet ($\alpha_i = 0$, $\beta_i = 1$), Neumann ($\alpha_i = 1$, $\beta_i = 0$) and Robin condition ($\alpha_i = 1$, $\beta_i > 0$). The initial function $\xi_i(x)$ is assumed to be positive in Ω and satisfies the boundary condition in (1.1). The above system describes the interaction among the three population species (u, v, w) where the u-species is a prey, the v-species is a predator on u, and w-species preys on v. This type of one-prey two-predator model is often referred to as simple food-chain model (cf. [1,6,12,13]). The term $\Delta u_i^{m_i}$ with $m_i > 1$ for $(u_1, u_2, u_3) = (u, v, w)$ implies that the diffusion rate of the u_i-species from high density region to low density region is slow and it models a tenancy of avoid crowding (cf. [17,23]).

To investigate the dynamics of the system (1.1) we need to consider the corresponding steady-state problem which is given by the quasilinear elliptic system

$$\begin{aligned}
&-d_1(x)\,\Delta u^{m_1} = u\,[a_1 - b_1 u - c_1 v/(u+\mu_1 v)] \\
&-d_2(x)\,\Delta v^{m_2} = v\,[-a_2 + b_2 u/(u+\mu_2 v) - c_2 w/(\mu_2' v + w)] \qquad (x\in\Omega) \\
&-d_3(x)\,\Delta w^{m_3} = w\,[-a_3 + b_3 v/(\mu_3 v + w)] \\
&\alpha_1 \partial u/\partial\nu + \beta_1 u = 0, \quad \alpha_2 \partial v/\partial\nu + \beta_2 v = 0, \quad \alpha_3 \partial w/\partial\nu + \beta_3 w = 0 \quad (x\in\partial\Omega) \qquad (1.2)
\end{aligned}$$

Since Problem (1.1) may be considered as a density-dependent reaction diffusion system with the density-dependent coefficients $D_i(u_i) = m_i u_i^{m_i - 1}$ this system is degenerate on the boundary when $m_i > 1$.

By allowing $c_2 = 0$ in (1.1) and (1.2) the time-dependent problem is reduced to a coupled system of one-prey one-predator problem in the form

$$\begin{aligned}
&\partial u/\partial t - d_1(x)\,\Delta u^{m_1} = u\,[a_1 - b_1 u - c_1 v/(u+\mu_1 v)] \\
&\partial v/\partial t - d_2(x)\,\Delta v^{m_2} = v\,[-a_2 + b_2 u/(u+\mu_2 v)] \qquad (t>0, \quad x\in\Omega) \\
&\alpha_1 \partial u/\partial\nu + \beta_1 u = 0, \quad \alpha_2 \partial v/\partial\nu + \beta_2 v = 0 \qquad (t>0, \quad x\in\partial\Omega) \\
&u(0,x) = \xi_1(x), \quad v(0,x) = \xi_2(x) \qquad (x\in\Omega) \qquad (1.3)
\end{aligned}$$

while the steady-state problem (1.2) becomes

$$\begin{aligned}
&-d_1(x)\,\Delta u^{m_1} = u\,[a_1 - b_1 u - c_1 v/(u+\mu_1 v)] \\
&-d_2(x)\,\Delta v^{m_2} = v\,[-a_2 + b_2 u/(u+\mu_2 v)] \qquad (x\in\Omega) \\
&\alpha_1 \partial u/\partial\nu + \beta_1 u = 0, \quad \alpha_2 \partial v/\partial\nu + \beta_2 v = 0 \qquad (x\in\partial\Omega) \qquad (1.4)
\end{aligned}$$

The above systems for the semilinear case $m_1 = m_2 = 1$ have also been investigated by many workers (cf. [3,4,9,16,15,21,22,25,27,26,29]). We shall deduce some conclusions for the system (1.3) and (1.4) from the discussion of the three-species models (1.1) and (1.2).