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Weak solutions of coupled dual porosity flows in fractured rock mass and structured porous media

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ABSTRACT

This paper deals with a fully nonlinear degenerate parabolic system with natural (critical) growths and under non-linear boundary conditions. Such problems arise from the heat and water flow through a partially saturated fractured rock mass and structured porous media. Existence of a global weak solution of the problem (on an arbitrary interval of time) is proved by means of semidiscretization in time, deriving suitable a-priori estimates based on $W^{1,p}$-regularity of the approximate solution and by passing to the limit from discrete approximations.

1. Introduction

In this paper we deal with mathematical analysis of fully nonlinear degenerate parabolic system modeling coupled heat transport and preferential movement of water in dual structured porous media. Variably-saturated porous medium is treated as a multi-phase material. At the microscale the individual phases can be clearly identified, however, at the macroscale, where measurements are usually carried out, the only observable quantities correspond to the effective behaviour. Because the detailed description of the geometry of the porous space is seldom known in practice, the macroscale-level equations are sought as suitable averages of the microscale balance law, for example in the framework of the hybrid mixture theory, originally proposed in [18–20]. In this context, the porous medium is considered as continuum of independent overlapping phases. For each constituent its conservation equation is derived according to principles of continuum mechanics.

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1.1. Conservation of mass

In mixture theory, the derivation of the mass balance equation is based on mass conservation of α-phase inside the spatial domain Ω of interest. A general form of a mass balance law reads [34]

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{B}} \varrho^{\alpha}\,\mathrm{d}x + \int_{\partial\mathcal{B}} \varrho^{\alpha}\boldsymbol{v}_{\alpha}\cdot\boldsymbol{n}\,\mathrm{d}S = \int_{\mathcal{B}} s_{\alpha}\,\mathrm{d}x \tag{1.1}$$

to be satisfied for any regular subdomain $\mathcal{B} \subset \overline{\mathcal{B}} \subset \Omega$. Here, $\varrho^{\alpha} = \Theta_{\alpha}\varrho_{\alpha}$ represents the phase averaged density, Θ_{α} [-] is the volume fraction of the α-phase, ϱ_{α} [kg m^{-3}] stands for the intrinsic phase averaged density and s_{α} [kg m^{-3} s^{-1}] is a production term. Further, $\boldsymbol{v}_{\alpha}$ [m s^{-1}] is the velocity of α-phase and $\boldsymbol{n}$ represents an outward unit normal vector to the boundary $\partial\mathcal{B}$. Applying the divergence theorem to (1.1) and owing to the arbitrariness of the domain $\mathcal{B}$ one arrives at the local form of the balance law

$$\frac{\partial(\Theta_{\alpha}\varrho_{\alpha})}{\partial t} + \nabla\cdot(\Theta_{\alpha}\varrho_{\alpha}\boldsymbol{v}_{\alpha}) = s_{\alpha}. \tag{1.2}$$

1.2. Conservation of heat energy

The balance of heat energy for the α-phase can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{B}} e_{\alpha}\,\mathrm{d}x + \int_{\partial\mathcal{B}} (\boldsymbol{q}_T)_{\alpha}\cdot\boldsymbol{n}\,\mathrm{d}S = \int_{\mathcal{B}} \mathcal{Q}_{\alpha}\,\mathrm{d}x + \int_{\mathcal{B}} \mathcal{E}_{\alpha}\,\mathrm{d}x - \int_{\mathcal{B}} H_{\alpha}s_{\alpha}\,\mathrm{d}x, \tag{1.3}$$

where e_{α} [J m^{-3}] is the total internal energy of the α-phase in $\mathcal{B}$, $(\boldsymbol{q}_T)_{\alpha}$ [W m^{-2}] is the heat flux, $\mathcal{Q}_{\alpha}$ stands for the volumetric heat source, $\mathcal{E}_{\alpha}$ represents the term expressing energy exchange with the other phases and the symbol H_{α} [J kg^{-1}] stands for the specific enthalpy of the α-phase. Here we assume

$$e_{\alpha} = \varrho^{\alpha}C_{\alpha}T_{\alpha}, \tag{1.4}$$

where T_{α} [K] is the absolute temperature and C_{α} [J kg^{-1} K^{-1}] represents the specific isobaric heat of the α-phase. Further, the heat flux vector $(\boldsymbol{q}_T)_{\alpha}$ includes the conductive flux $\boldsymbol{q}_{\alpha}$ and convection

$$(\boldsymbol{q}_T)_{\alpha} = \boldsymbol{q}_{\alpha} + \varrho^{\alpha}C_{\alpha}T_{\alpha}\boldsymbol{v}_{\alpha}. \tag{1.5}$$

Hence, applying the divergence theorem to (1.3) and using (1.4) and (1.5) one obtains the heat energy conservation equation for the α-phase in the differential form

$$\partial_t\left(\varrho^{\alpha}C_{\alpha}T_{\alpha}\right) + \nabla\cdot\left(\boldsymbol{q}_{\alpha} + \varrho^{\alpha}C_{\alpha}T_{\alpha}\boldsymbol{v}_{\alpha}\right) = \mathcal{Q}_{\alpha} + \mathcal{E}_{\alpha} - H_{\alpha}s_{\alpha}. \tag{1.6}$$

1.3. Single porosity model

In the simplest case, consider the flow of a single homogeneous fluid through a porous solid, such as variably saturated water flow in soils. The mass conservation equation for the α-phase (1.2) can be particularized to both the water phase ($\alpha = w$) and the solid phase ($\alpha = s$). The mass conservation equations for the water and solid phases, respectively, become (neglecting source terms)

$$\frac{\partial(\Theta_{w}\varrho_{w})}{\partial t} + \nabla\cdot(\Theta_{w}\varrho_{w}\boldsymbol{v}_{w}) = 0 \tag{1.7}$$