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Variational minimizing parabolic orbits for 2-fixed center problems

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ABSTRACT

Using variational minimizing methods, we prove the existence of an odd symmetric parabolic orbit for 2-fixed center problems with weak force type homogeneous potentials.

1. Introduction and main results

The circular restricted 3-body problem has attracted many researchers, e.g., Sitninkov [13], Moser [11], Mathlouthi [9], Souissi [14], and Zhang [16]. In this problem, it appears that two bodies with equal mass ($m_1 = m_2 = 1/2$) move in a circular orbit in a plane where their center of mass is at the origin. The motion of a third massless body is then considered under the attraction of the first two bodies. However, the circular motion of the first two bodies is not influenced by the third massless body. In particular, the massless body can move in a straight line perpendicular to the circular orbit plane and through the center of mass of the first two bodies.

Let $z(t)$ be the coordinate of the third body. Then, $z(t)$ satisfies

$$\ddot{z}(t) + \alpha \frac{z(t)}{(|z(t)|^2 + |r|^2)^{\alpha/2+1}} = 0. \tag{1}$$

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Zhang [16] used the variational minimizing method to prove the existence of an odd parabolic or hyperbolic orbit for Equation (1) with $0 < \alpha < 2$.

In this study, we consider the 2-fixed center problem, which is a classical problem with a long history [3–5,1,7,8,15]. For two masses, $1-\mu$ and μ fixed at $q_1 = (-\mu, 0)$ and $q_2 = (1-\mu, 0)$, respectively, the problem involves studying the motion $q(t) = (x(t), y(t))$ of a third body with mass $m_3 > 0$. In the present study, we consider that the motion of the third body is attracted by 2-fixed center masses with general homogeneous potentials, which satisfies the following equation:

$$\ddot{q}(t) + \frac{\partial V(q)}{\partial q} = 0, \tag{2}$$

$$V(q) = -\frac{1-\mu}{|q-q_1|^\alpha} - \frac{\mu}{|q-q_2|^\alpha}. \tag{3}$$

Definition 1.1. We refer to the solution $\tilde{q}_\alpha(t)$ of (2)–(3) as a parabolic type solution if

$$\max_{t\in R} |\tilde{q}_\alpha(t)| = +\infty,$$
$$\min_{t\in R} |\dot{\tilde{q}}_\alpha(t)| = 0$$

and the energy along the solution is zero:

$$\frac{1}{2}|\dot{\tilde{q}}_\alpha|^2 - \frac{1-\mu}{|\tilde{q}_\alpha - q_1|^\alpha} - \frac{\mu}{|\tilde{q}_\alpha - q_2|^\alpha} = h = 0. \tag{4}$$

For $\mu = 1/2$, we consider the existence of the motion $q(t) = (x(t), y(t))$ of the third body, which satisfies the odd symmetry $(x(-t), y(-t)) = (-x(t), -y(t))$. We use the variational minimizing method to prove the following.

Theorem 1.1. *For (2)–(3) with $\mu = \frac{1}{2}$ and $0 < \alpha < 2$, an odd symmetrical parabolic-type solution exists.*

Remark. Note that [16] studied the existence of the parabolic or hyperbolic solution for restricted 3-body problems, but the author only proved the energy $h \geq 0$ along the unbounded solution, whereas we provide a more detailed analysis to determine the existence of the parabolic solution for 2-fixed center problems, and thus we prove the energy $h = 0$ along the unbounded solution. We also note that the potentials for these two problems are similar but not the same.

2. Truncation functional and its minimizing critical points

In order to determine the parabolic-type orbit of (2)–(3), we first restrict $t \in [-n, n]$ and find solutions of (2)–(3), and we then let $n \to +\infty$ to obtain the parabolic-type orbit. By noting the symmetry of the equation, we can find the odd solutions of the following ODE:

$$\ddot{q}(t) = \frac{\partial U(q)}{\partial q}, \tag{5}$$

$$U(q) = \frac{1/2}{|q-q_1|^\alpha} + \frac{1/2}{|q-q_2|^\alpha}. \tag{6}$$

We define the functional:

$$f(q) = \int_{-n}^{n} (\frac{1}{2}|\dot{q}(t)|^2 + \frac{1/2}{|q-q_1|^\alpha} + \frac{1/2}{|q-q_2|^\alpha})dt, \tag{7}$$