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On the minimum eccentric distance sum of bipartite graphs with some given parameters ☆

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ABSTRACT

The eccentric distance sum is a novel graph invariant with vast potential in structure activity/property relationships. This graph invariant displays high discriminating power with respect to both biological activity and physical properties. If $G = (V_G, E_G)$ is a simple connected graph, then the eccentric distance sum (EDS) of G is defined as $\xi^d(G) = \sum_{v \in V_G} \varepsilon_G(v) D_G(v)$, where $\varepsilon_G(v)$ is the eccentricity of the vertex v and $D_G(v) = \sum_{u \in V_G} d_G(u, v)$ is the sum of all distances from the vertex v. Much extremal work has been done on trees with some given parameters by Yu et al. (2011) [25], Li et al. (2012) [19], and Geng et al. (2013) [6]. It is natural to consider this extremal problem on bipartite graphs with some given parameters. In this paper, sharp lower bound on the EDS in the class of all connected bipartite graphs with a given matching number q is determined, the minimum EDS is realized only by the graph $K_{q,n-q}$. The extremal graph with the minimum EDS in the class of all the n-vertex connected bipartite graphs of odd diameter is characterized. All the extremal graphs having the minimum EDS in the class of all connected n-vertex bipartite graphs with a given vertex connectivity are identified as well.

1. Introduction

In this paper, we consider connected, simple and undirected graphs. We follow the notation and terminology in [2] except if otherwise stated.

Let $G = (V_G, E_G)$ be a graph with $u, v \in V_G$. Then $G - v$, $G - uv$ denote the graph obtained from G by deleting vertex $v \in V_G$, or edge $uv \in E_G$, respectively (this notation is naturally extended if more than one vertex or edge is deleted). Similarly, $G + uv$ is obtained from G by adding an edge $uv \notin E_G$. The *distance*, $d_G(u, v)$ between two vertices u, v of G is the length of a shortest u–v path in G. The *eccentricity* $\varepsilon_G(v)$

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of a vertex v is the distance between v and a furthest vertex from v. The *diameter* of G is defined as the maximum of the eccentricities of vertices of G, whereas the *radius* of G is the minimum of the eccentricities of vertices of G.

The *join* $G_1 + G_2$ of two vertex disjoint graphs G_1 and G_2 is the graph consisting of the union $G_1 \cup G_2$, together with all edges of the type xy, where $x \in V_{G_1}$ and $y \in V_{G_2}$. For $k \geqslant 3$ vertex disjoint graphs $G_1, G_2, \ldots, G_k$, the *sequential join* $G_1+G_2+\cdots+G_k$ is the graph $(G_1+G_2)\cup(G_2+G_3)\cup\cdots\cup(G_{k-1}+G_k)$. The sequential join of k disjoint copies of a graph G will be denoted by $[k]G$, the union of k disjoint copies of G will be denoted by kG, while $[s]G_1 + G_2 + [t]G_3$ will denote the sequential join $\underbrace{G_1 + G_1 + \cdots + G_1}_{s} + G_2 + \underbrace{G_3 + G_3 + \cdots + G_3}_{t}$.

A *bipartite graph* G is a simple graph, whose vertex set V_G can be partitioned into two disjoint subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. A bipartite graph in which every two vertices from different partition classes are adjacent is called *complete*, which is denoted by $K_{m,n}$, where $m = |V_1|$, $n = |V_2|$. For convenience, let $\mathscr{A}_n^q$ be the class of all connected bipartite graphs of order n with matching number q; $\mathscr{B}_n^d$ be the class of all connected bipartite graphs of order n with diameter d; $\mathscr{C}_n^s$ be the class of n-vertex connected bipartite graphs with connectivity s.

For a vertex subset S of V_G, denoted by $G[S]$ the subgraph induced by S. Denote by P_n and K_n, the path and complete graph on n vertices, respectively. For a real number x we denote by $\lfloor x \rfloor$ the greatest integer $\leqslant x$, and by $\lceil x \rceil$ the least integer $\geqslant x$.

A graph G is called *k-connected* if $|V_G| > k$ and $G - X$ is connected for every set $X \subseteq V_G$ with $|X| < k$. The connectivity $\kappa(G)$ of G is the maximum value of k for which G is k-connected. G is 1-connected if and only if it is connected; equivalently, $\kappa(G) = 0$ if and only if it is disconnected.

A *vertex* (*edge*) *independent set* of a graph G is a set of vertices (edges) such that any two distinct vertices (edges) of the set are not adjacent (incident on a common vertex). A *vertex* (*edge*) *independent number* of G, denoted by $\alpha(G)$ ($\alpha'(G)$) is the maximum of the cardinalities of all vertex (edge) independent sets. A *vertex* (*edge*) *cover* of a graph G is a set of vertices (edges) such that each edge (vertex) of G is incident with at least one vertex (edge) of the set. A *vertex* (*edge*) *cover number* of G, denoted by $\beta(G)$ ($\beta'(G)$) is the minimum of the cardinalities of all vertex (edge) covers. When we consider an edge cover of a graph, we always assume that the graph contains no vertices whose degree is 0. It is known that for a graph G of order n, $\alpha(G) + \beta(G) = n$ and $\alpha'(G) + \beta'(G) = n$. For a bipartite graph, we have $\alpha'(G) = \beta(G)$ and $\alpha(G) = \beta'(G)$.

The study of distances between vertices of a tree probably started from the classic *Wiener index* [24], which is one of the most well used chemical indices that correlate a chemical compound's structure (the "molecular graph") with the compound's physical–chemical properties. The Wiener index, introduced in 1947, is defined as the sum of distances between all pairs of vertices, namely that

$$W(G) = \sum_{\{u,v\}\subseteq V_G} d_G(u, v).$$

For more results on Wiener index one may be referred to those in [4,10,12,17] and the references cited therein.

A weighted graph (G, w) is a graph $G = (V_G, E_G)$ together with the weight function $w : V_G \to \mathbb{N}^+$. The Wiener index $W(G, w)$ of (G, w) is then defined as

$$W(G, w) = \frac{1}{2} \sum_{u\in V_G} \sum_{v\in V_G} (w(u) \oplus w(v)) d_G(u, v), \tag{1.1}$$