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Stability estimates for the inverse boundary value problem for the biharmonic operator with bounded potentials

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ABSTRACT

In this article, stability estimates are given for the determination of the zeroth-order bounded perturbations of the biharmonic operator when the boundary Neumann measurements are made on the whole boundary and on slightly more than half the boundary, respectively. For the case of measurements on the whole boundary, the stability estimates are of ln-type and for the case of measurements on slightly more than half of the boundary, we derive estimates that are of ln ln-type.

1. Introduction

Let $\Omega \subset \mathbb{R}^n$, $n \geq 3$ be a bounded domain with C^∞ boundary and consider the following equation:

$$\mathcal{B}_q u := (\Delta^2 + q)u = 0 \text{ in } \Omega, \quad q \in L^\infty(\Omega).$$

We consider the following space for the potential q:

$$\mathcal{Q}_M := \{q : \text{ supp}(q) \subset \overline{\Omega}, \text{ and } \|q\|_{L^\infty(\Omega)} \leq M \text{ for some } M > 0\}. \tag{1}$$

We will assume that for all $q \in \mathcal{Q}_M$, 0 is not an eigenvalue for $\mathcal{B}_q$ on the set $\{u \in H^4(\Omega) : u|_{\partial\Omega} = \Delta u|_{\partial\Omega} = 0\}$. Then given $(f, g) \in H^{7/2}(\partial\Omega) \times H^{3/2}(\partial\Omega)$, there is a unique solution to the boundary value problem:

$$\mathcal{B}_q u = 0, \quad u|_{\partial\Omega} = f, \quad \Delta u|_{\partial\Omega} = g. \tag{2}$$

The boundary conditions are called Navier conditions [6] and we define the Dirichlet-to-Neumann map $\mathcal{N}_q$ for this operator by

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$$\mathcal{N}_q : H^{7/2}(\partial\Omega) \times H^{3/2}(\partial\Omega) \to H^{5/2}(\partial\Omega) \times H^{1/2}(\partial\Omega)$$
$$(f,g) \to (\frac{\partial u}{\partial \nu}|_{\partial\Omega}, \frac{\partial(\Delta u)}{\partial \nu}|_{\partial\Omega}), \tag{3}$$

where $u \in H^4(\Omega)$ is the unique solution to (2).

We are interested in the inverse problem of determining q from $\mathcal{N}_q$. The uniqueness question of determination of q from $\mathcal{N}_q$ was answered in [9,10] and recently in [11,12,17] where they showed that unique determination of both zeroth- and first-order perturbations of the biharmonic operator is possible from boundary Neumann data. We note that the papers [11,17] also show unique determination of the first-order perturbation terms from Neumann data measured on possibly small subsets of the boundary.

In this paper, we consider the stability question for the determination of q from $\mathcal{N}_q$ for the operator $\mathcal{B}_q$. That is, whether one can estimate perturbations of q from perturbations of the Neumann data $\mathcal{N}_q$. To the best of the authors' knowledge, stability estimates for inverse problems involving the biharmonic equation has not been obtained earlier, and the purpose of this paper is to investigate it. We prove a stability estimate of ln-type for the case when the Neumann data is measured on the whole boundary. We then prove a stability estimate of ln ln-type when the Neumann data is measured on a part of the boundary that is slightly more than half the boundary.

Our strategy for proving stability estimates follows the methods introduced by Alessandrini in [1] using complex geometric optics (CGO) solutions where a ln-type stability estimate is proved for the Calderón inverse problem [3], and by Heck and Wang in [8] where a ln ln-type stability estimate is proved for the Calderón inverse problem when the Neumann data is measured on slightly more than half of the boundary. CGO solutions were introduced by Sylvester and Uhlmann in the fundamental paper [14] to prove global uniqueness for the Calderón inverse problem. The method in Heck and Wang combines CGO solutions and techniques of [2] with an analytic continuation result of Vessella [16]. Stability estimates for several inverse problems have been obtained in recent years. Apart from the works [1,8] already mentioned, we refer the reader to [15,7,5,4] for stability estimates involving the Calderón inverse problem and inverse problems involving the Schrödinger or magnetic Schrödinger equation.

2. Statements of the main results

We now state the main results of this paper. We first consider stability estimates for full boundary measurements and then prove stability estimates when only partial boundary measurements are available.

2.1. Results for full boundary measurements

Consider the following norm on $H^\alpha(\partial\Omega) \times H^\beta(\partial\Omega)$ (for simplicity we will denote this space by $H^{\alpha,\beta}(\partial\Omega)$):

$$\|(f,g)\|_{H^{\alpha,\beta}(\partial\Omega)} = \|f\|_{H^\alpha(\partial\Omega)} + \|g\|_{H^\beta(\partial\Omega)} \text{ for } (f,g) \in H^{\alpha,\beta}(\partial\Omega). \tag{4}$$

Define:

$$\|\mathcal{N}_q\| = \sup\{\|\mathcal{N}_q(f,g)\|_{H^{\frac{5}{2},\frac{1}{2}}(\partial\Omega)} : \|(f,g)\|_{H^{\frac{7}{2},\frac{3}{2}}(\partial\Omega)} = 1\}$$

where $\mathcal{N}_q(f,g)$ is defined in (3).

Theorem 2.1. *Let $\Omega \subset \mathbb{R}^n$, $n \geq 3$ be a bounded domain with smooth boundary. Consider Equation (2) for two potentials $q_1, q_2 \in \mathcal{Q}$. Let $\mathcal{N}_{q_1}$ and $\mathcal{N}_{q_2}$ be the corresponding Dirichlet-to-Neumann maps measured on $\partial\Omega$.*