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Boundary value problems for discontinuous perturbations of singular ϕ-Laplacian operator

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ABSTRACT

Systems of differential inclusions of the form

$$-(\phi(u'))' \in \partial F(t,u), \quad t \in [0,T],$$

where $\phi = \nabla\Phi$, with Φ strictly convex, is a homeomorphism of the ball $B_a \subset \mathbb{R}^N$ onto $\mathbb{R}^N$, are considered under Dirichlet, periodic and Neumann boundary conditions. Here, $\partial F(t,x)$ stands for the generalized Clarke gradient of $F(t,\cdot)$ at $x \in \mathbb{R}^N$. Using nonsmooth critical point theory, we obtain existence results under some appropriate conditions on the potential F. Examples of applications concerning Filippov type solutions are also provided.

1. Introduction

Using a variational approach, we obtain existence results for differential inclusions systems of type

$$-(\phi(u'))' \in \partial F(t,u), \quad t \in [0,T]; \qquad l(u,u') = 0, \tag{1.1}$$

where $l(u,u') = 0$ denotes one of the Dirichlet, periodic or Neumann boundary conditions and ϕ satisfies

(H_Φ) *ϕ is a homeomorphism from B_a onto $\mathbb{R}^N$ such that $\phi(0) = 0$, $\phi = \nabla\Phi$, with $\Phi : \overline{B}_a \to (-\infty, 0]$ of class C^1 on B_a, continuous and strictly convex on $\overline{B}_a$.*

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The mapping $F : [0, T] \times \mathbb{R}^N \to \mathbb{R}$ is assumed to verify the hypothesis:

(H_F) *(i)* $F(\cdot, x) : [0, T] \to \mathbb{R}$ *is measurable for every* $x \in \mathbb{R}^N$ *and* $F(\cdot, 0) = 0$;
(ii) for each $\rho > 0$ *there is some* $\alpha_\rho \in L^1([0, T]; \mathbb{R})$ *such that, for all* $t \in [0, T]$ *and* $x, y \in \overline{B}_\rho$, *it holds*

$$|F(t, x) - F(t, y)| \leq \alpha_\rho(t)|x - y|.$$

Here and hereafter, $|\cdot|$ stands for the Euclidean norm on $\mathbb{R}^N$, $B_\sigma \subset \mathbb{R}^N$ denotes the open ball of center 0 and radius σ and $\partial F(t, x)$ stands for the generalized Clarke gradient of $F(t, \cdot)$ at $x \in \mathbb{R}^N$. Also, throughout the paper, we denote by C the space of all continuous functions $u : [0, T] \to \mathbb{R}^N$, endowed with the usual norm $\|u\|_C = \max_{t\in[0,T]} |u(t)|$.

A function $u \in C^1 := C^1([0, T]; \mathbb{R}^N)$ is said to be *solution* of the differential inclusions system in problem (1.1) if $\|u'\|_C < a$, $\phi(u')$ is absolutely continuous and u satisfies

$$-(\phi(u'(t)))' \in \partial F(t, u(t)), \quad \text{for a.e. } t \in [0, T].$$

For $p \in (1, \infty)$, let ϕ_p be defined by

$$\phi_p(y) = \frac{|y|^{p-2}y}{(1 - |y|^p)^{1-1/p}} \quad (y \in B_1).$$

It is easy to see that ϕ_p satisfies (H_Φ) with $\Phi_p : \overline{B}_1 \to \mathbb{R}$ given by

$$\Phi_p(y) = -(1 - |y|^p)^{1/p} \quad (y \in \overline{B}_1).$$

This choice of ϕ engenders the "*p-relativistic*" operator

$$u \mapsto \mathcal{R}_p u := \left(\frac{|u'|^{p-2}u'}{(1 - |u'|^p)^{1-1/p}}\right)';$$

notice that the classical *relativistic* operator is obtained for $p = 2$.

In recent years, much attention has been paid to the study of boundary value problems with singular ϕ-Laplacian, with special attention to the relativistic operator $\mathcal{R}_2$ (see e.g., [3,5,6,9,16,19] and the references therein). Mainly, the obtained results concern the existence and the multiplicity of solutions for problems involving continuous perturbations of the ϕ-Laplacian and less of them deal with discontinuous perturbations. In this second direction, we refer to the papers [10] and [23], where the existence of solutions of differential inclusions systems is derived by means of fixed-point and topological techniques. In [17] the variational method is employed to treat the periodic problem

$$-\mathcal{R}_2 u \in \partial F(t, u), \qquad u(0) - u(T) = 0 = u'(0) - u'(T), \tag{1.2}$$

with $F : [0, T] \times \mathbb{R}^N \to \mathbb{R}$ satisfying (H_F) (i) together with

$(\widetilde{H}_F)$ *there exists* $\alpha \in L^1([0, T]; \mathbb{R})$ *such that for all* $t \in [0, T]$ *and* $x, y \in \mathbb{R}^N$, *it holds*

$$|F(t, x) - F(t, y)| \leq \alpha(t)|x - y|,$$