J. Math. Anal. Appl. 433 (2016) 803–817

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Some extremal results on the connective eccentricity index of graphs [☆]

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A R T I C L E   I N F O

Article history:
Received 27 May 2015
Available online 12 August 2015
Submitted by S.A. Fulling

Keywords:
Connective eccentricity index
Eccentric connectivity index
Matching number
Tree
Extremal graph

A B S T R A C T

The connective eccentricity index (CEI) of a graph G is defined as $\xi^{ce}(G) = \sum_{v_i \in V(G)} \frac{d(v_i)}{\varepsilon(v_i)}$ where $\varepsilon(v_i)$ and $d(v_i)$ are the eccentricity and the degree of vertex v_i, respectively, in G. In this paper we obtain some lower and upper bounds on the connective eccentricity index for all trees of order n and with matching number β and characterize the corresponding extremal trees. And the maximal graphs of order n and with matching number β and n edges have been determined which maximize the connective eccentricity index. Also the extremal graphs with maximal connective eccentricity index are completely characterized among all connected graphs of order n and with matching number β. Moreover we establish some relations between connective eccentricity index and eccentric connectivity index, as another eccentricity-based invariant, of graphs.

1. Introduction

We only consider finite, undirected and simple graphs throughout this paper. Let G be a graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. For any vertex $v_i \in V(G)$, let $N_G(v_i)$ be the set of neighbors of v_i (i.e., vertices adjacent to v_i) in G, the *degree* of $v_i \in V(G)$, denoted by $d_G(v_i)$, is the cardinality of $N_G(v_i)$. In particular, the maximum and minimum degree of a graph G will be denoted by $\Delta(G)$ and $\delta(G)$, respectively. And a vertex v_i of degree 1 is called a *pendant vertex*, the edge incident with pendant vertex is called a *pendant edge*. For two vertices v_i, $v_j \in V(G)$, the distance between v_i and v_j,

☆ The first author is supported by NNSF of China (No. 11201227), China Postdoctoral Science Foundation (2013M530253, 2014T70512) and Natural Science Foundation of Jiangsu Province (BK20131357). The second author is supported by the National Research Foundation funded by the Korean government with the grant No. 2013R1A1A2009341.

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http://dx.doi.org/10.1016/j.jmaa.2015.08.027
0022-247X/

denoted by $d_G(v_i, v_j)$, is the length (i.e., the number of edges) of a shortest path connecting them in G. For a vertex $v_i \in V(G)$, the *eccentricity* of v_i, denoted by $\varepsilon_G(v_i)$, is the maximum distance from v_i to other vertices in G. The *diameter* of a graph G, denoted by $d(G)$, is the maximum eccentricity among all vertices in G, and the *radius* of G, denoted by $r(G)$, is the minimum of all eccentricities of vertices in G. In what follows, if no ambiguity occurs, the subscript G will be omitted in $d_G(v_i)$, $d_G(v_i, v_j)$ and $\varepsilon_G(v_i)$. For a subset W of $V(G)$, let $G - W$ be the subgraph of G obtained by deleting the vertices of W and the edges incident with them. Similarly, for a subset E' of $E(G)$, we denote by $G - E'$ the subgraph of G obtained by deleting the edges of E'. If $W = \{v_i\}$ and $E' = \{v_j v_k\}$, the subgraphs $G - W$ and $G - E'$ will be written as $G - v_i$ and $G - v_j v_k$ for short, respectively. For any two nonadjacent vertices v_p and v_q of a graph G, we let $G + v_p v_q$ be the graph obtained from G by adding a new edge $v_p v_q$. Throughout this paper we use P_n, S_n, C_n and K_n to denote the path graph, star graph, cycle graph and complete graph on n vertices, respectively. Other undefined notations and terminology on the graph theory can be found in [4].

In chemical graph theory, various graphical invariants are used for establishing correlations of chemical structures with various physical properties, chemical reactivity, or biological activity. Among these *topological indices*, as they are called in the area, there is a large family of ones based on the distance, including eccentricity and general distance, in a graph. In 2000, Gupta et al. [13] introduced a novel, adjacency-cum-path length based, topological descriptor named as connective eccentricity index (CEI) when investigating the antihypertensive activity of derivatives of N-benzylimidazole. They showed that the results obtained using the connective eccentricity index were better than the corresponding values obtained using Balaban's mean square distance index [1,2] and the accuracy of prediction was found to be about 80 percents in the active range [13]. Moreover, Sharma et al. [25] introduced the eccentric connectivity index, for a graph, which has been employed successfully for the development of numerous mathematical models for the prediction of biological activities of diverse nature [12,24,25]. Some nice results on other attractive distance-based topological indices of graphs can be found in [14,16,19,31] for eccentric distance sum, [6] for Zagreb eccentricity indices, [17] for average eccentricity, [23] for adjacent eccentric distance sum index, [10,18,26,27] for degree distance, [11,15] for Gutman index and a recent survey [29] for extremal problems. Furthermore, some interesting properties of eccentricity are reported in [20,21].

For a graph G, the connective eccentricity index $\xi^{ce}(G)$ [13] and the eccentric connectivity index (ECI) $\xi^c(G)$ [25] are defined, respectively, in the following:

$$\xi^{ce}(G) = \sum_{v_i \in V(G)} \frac{d(v_i)}{\varepsilon(v_i)}\,,$$

$$\xi^{c}(G) = \sum_{v_i \in V(G)} d(v_i)\varepsilon(v_i)\,.$$

Alternatively, from their respective definitions, we have

$$\xi^{ce}(G) = \sum_{v_i v_j \in E(G)} \Big(\frac{1}{\varepsilon(v_i)} + \frac{1}{\varepsilon(v_j)}\Big), \tag{1}$$

$$\xi^{c}(G) = \sum_{v_i v_j \in E(G)} \Big(\varepsilon(v_i) + \varepsilon(v_j)\Big). \tag{2}$$

Some recent results on the connective eccentricity index and the eccentric connectivity index of graphs can be found in [22,30,32–34].

Two edges in a graph G are called *independent* if they are not incident with one common vertex in it. A *matching* of a graph G is a subset of mutually independent edges of G. For a graph G, the *matching number* $\beta(G)$ is the maximum cardinality of all the matchings in G. If M is a matching of a graph G and a vertex