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On the optimal control problem for the Novikov equation with strong viscosity

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ABSTRACT

In this paper, we consider an optimal control problem for the Novikov equation with strong viscosity. Using the Faedo–Galerkin method we derive the existence of a unique weak solution to this equation. Applying Lions' theory, we obtain the existence of an optimal solution to the control problem for this equation. We also deduce the first-order necessary optimality condition. Moreover we establish two second-order sufficient optimality conditions, which require coercivity of the augmented Lagrangian functional on the whole space or on a suitable subspace.

1. Introduction

The Novikov equation

$$y_t + u^2 y_x + 3uu_x y = 0, \quad y = u - u_{xx}, \tag{1.1}$$

was isolated by Novikov [22] in a symmetry classification of nonlocal partial differential equations. Compared with the well-studied Camassa–Holm equation [2]

$$y_t + uy_x + 2u_x y = 0, \quad y = u - u_{xx}, \tag{1.2}$$

the Novikov equation has nonlinear terms that are cubic, rather than quadratic, which can be thought as a generalization of the Camassa–Holm equation. Novikov [22] found its first few symmetries and he subsequently found a scalar Lax pair for it, proving that the equation is integrable. Hone and Wang [11] gave

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a matrix Lax pair for the Novikov equation, and showed how it was related by a reciprocal transformation to a negative flow in the Sawada–Kotera hierarchy. Infinitely many conserved quantities were found, as well as a bi-Hamiltonian structure. They also presented peakons for Eq. (1.1). Liu, Liu and Qu [20] proved such peakons are orbital stable. Hone, Lundmark and Szmigielski [12] calculated the explicit formulas for multipeakon solutions of Eq. (1.1), using the matrix Lax pair found by Hone and Wang. Very recent works are intensively devoted to studying the local well-posedness, global existence of strong and weak solution, and blow up of solution of initial value problem for Eq. (1.1) in Sobolev spaces or Besov spaces [32,21,44,45,8,46,14,17].

In general, the energy dissipation mechanisms are difficult to avoid in a real world. Many authors considered the dissipative effects on the practical models. For example, Ott and Sudan [24] investigated the KdV equation with the presence of dissipation and their effect on solution of the KdV equation. The long time behavior of solutions to the weakly dissipative KdV equation was studied by Ghidaglia [7]. Recently, Wu and Yin investigated the weakly dissipative Camassa–Holm equation

$$y_t + uy_x + 2u_x y + \lambda y = 0, \quad y = u - u_{xx}, \tag{1.3}$$

on the line [43] and on the circle [40]. They also studied the weakly dissipative Degasperis–Procesi equation

$$y_t + uy_x + 3u_x y + \lambda y = 0, \quad y = u - u_{xx}, \tag{1.4}$$

on the line [41,39] and on the circle [42], where $\lambda > 0$ denotes the weak viscosity. In [47], Yan, Li and Zhang considered the weakly dissipative Novikov equation

$$y_t + u^2 y_x + 3uu_x y + \lambda y = 0, \quad y = u - u_{xx}, \tag{1.5}$$

and obtained the global existence and blow-up phenomenon for this equation. However, it has been pointed out in [18] that these weakly dissipative equations (1.3), (1.4) and (1.5) are equivalent to the original equations up to a trivial change of variables, so that nothing new is gained by considering the weakly dissipative version.

On the other hand, the optimal control of viscous partial differential equation (PDE) are widely investigated. Let us mention some papers concerning this issue. Ghattas and Bark [6] studied the optimal control of two and three dimensional incompressible Navier–Stokes flows. Vedantham [34] developed a technique to utilize the Cole–Hopf transformation to solve an optimal control problem for the Burgers equation. Sang-Uk Ryu and Atsushi Yagi [26] investigated the optimal control of the Keller–Segel equations. Volkwein [36] used the augmented Lagrangian–SQP technique to solve the optimal control problem governed by the Burgers equation. Hinze and Volkwein [10] discussed the instantaneous control of the Burgers equation. Lagnese and Leugering [16] considered the problem of boundary optimal control of a wave equation with boundary dissipation. Oksendal [23] proved a sufficient maximum principle for the optimal control systems described by a quasi-linear stochastic heat equation. Based on the energy estimates and the compact method, Tian, Shen et al. studied the optimal control problems for the viscous Camassa–Holm equation, viscous Degasperis–Procesi equation and viscous Dullin–Gottwalld–Holm equation [31,30,27]. Under boundary condition, Zhao and Liu [48] studied the optimal control problem for viscous Cahn–Hilliard equation.

It is known that sufficient conditions form a central issue for mathematical different questions of optimal control theory and turned out to be indispensable for a complete numerical analysis of optimal control problems. Second-order conditions were investigated in many research papers on optimal control theory of PDEs. The elliptic equations were studied by Casas, Tröltzsch and Unger [3]. Rösch and Tröltzsch [25] considered the sufficient second-order optimality conditions for a parabolic optimal control problem. Volkwein investigated the stationary Burgers [35] and instationary Burgers equation [36]. Tröltzsch and Wachsmuth [33] discussed second-order sufficient optimality conditions for optimal control problems governed by steady-state