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New criteria on persistence in mean and extinction for stochastic competitive Lotka–Volterra systems with regime switching

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ABSTRACT

This paper concerns persistence in mean and extinction for stochastic competitive Lotka–Volterra systems with regime switching. By using some novel stochastic analysis techniques, sufficient criteria for the partial permanence and partial extinction are established. Some novel sufficient conditions on persistence in mean and extinction are also obtained. Nontrivial examples are provided to illustrate our results.

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1. Introduction

One of the most common phenomena considering ecological population is that many species which grow in the same environment compete for the limited resources or in some way inhibit others' growth. It is therefore very important to study the competition models for multi-species. It is well known that one of the most famous models is the following classical Lotka–Volterra competition system

$$\frac{dx_i}{dt} = x_i(b_i - \sum_{j=1}^{n} a_{ij}x_j), \quad i = 1, \cdots, n, \tag{1}$$

where $x_i(t)$ represents the population size of species i at time t, the constant b_i is the growth rate of species i, and a_{ij} represents the effect of interspecific ($i \neq j$) or intraspecific ($i = j$) interaction.

On the other hand, population systems are inevitably affected by environmental noise. One type of environmental noise is color noise, say telegraph noise. The telegraph noise can be illustrated as a switching

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between two or more regimes of environment, which differs by factors such as nutrition or precipitation. The switching is memoryless and the waiting time for the next switch has an exponential distribution. We can hence model the regime switching by a finite-state Markovian chain. The other is the well-know white noise described by the Brownian motion. In recent years, the population dynamics under environmental noise have been considered by many authors (see [3,4,6–11]).

When taking the white and color noise into account, system (1) becomes the stochastic competitive Lotka–Volterra system with regime switching as the following:

$$dx_i = x_i(b_i(r(t)) - \sum_{j=1} a_{ij}(r(t))x_j)dt + \sigma_i(r(t))x_i dB_i(t), \quad i = 1, \cdots, n. \tag{2}$$

The stochastic competitive Lotka–Volterra model has been extensively studied due to its universal existence and importance (see [1,2,5,6,15,16]). In the study of population systems, extinction and permanence, including stochastic permanence, persistence in mean, are two important and interesting properties, respectively meaning that the population system will die out or survive in the future, that have received a lot of attention (see [2–6]). Li et al. [6] discussed the stochastic general Lotka–Volterra population under regime switching, and sufficient conditions for stochastic permanence and extinction were obtained. Li et al. [4] discussed the stochastic logistic population under regime switching, and sufficient and necessary conditions for stochastic permanence and extinction under some assumption were obtained. More recently, a class of non-autonomous stochastic Lotka–Volterra competitive system was discussed by Li and Mao [5], the sufficient conditions for the stochastic permanence, extinction were obtained. Jiang et al. [2] have studied the stable in time average of the autonomous stochastic Lotka–Volterra competitive system, which implied the persistence in mean.

However, most of the existing criteria are established for stochastic general Lotka–Volterra system. It is well known that these criteria will produce conservatism when dealing with the competitive systems. And there are a few results on the persistence in mean for competitive systems available in the literature. This motivates us to investigate the persistence in mean and extinction for stochastic competitive Lotka–Volterra systems with regime switching.

Moreover, most of the existing criteria are established for total extinction and total permanence. The partial permanence and partial extinction, which are very important and useful properties, have not been fully investigated. To the best of our knowledge, results on this problem are rare, that remains an interesting research topic.

We aim to establish new results on persistence in mean and extinction for system (2). By using the Lyapunov methods, and some novel stochastic analysis techniques, sufficient criteria are established which ensure the partial permanence and partial extinction. Sufficient conditions on persistence in mean and extinction for system (2) are also obtained.

2. Notation

Throughout this paper, unless otherwise specified, let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is increasing and right continuous while $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets). Let $B(t) = (B_t^1, \ldots, B_t^d)$ be a d-dimensional Brownian motion defined on the probability space. Let $r(t)$, $t \geq 0$, be a right-continuous Markovian chain on the probability space taking values in a finite state space $S = \{1, 2, \ldots, \mathrm{N}\}$ with generator $\Gamma = (\gamma_{ij})_{\mathrm{N}\times\mathrm{N}}$ given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & i \neq j \\ 1 + \gamma_{ij}\Delta + o(\Delta) & i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} > 0$ is transition rate from i to j if $i \neq j$ while $\gamma_{ii} = -\sum_{i\neq j} \gamma_{ij} < 0$. We assume that the Markovian chain $r(t)$ is independent of the Brownian motion $B(t)$. It is well known that almost

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