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Permanence and extinction for a single-species system with jump-diffusion [☆]

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ABSTRACT

In this study, we consider the effect of jump-diffusion random environmental perturbations on the permanence and extinction of a single-species dispersal periodic system in poor patchy environments with the possibility of species loss during their dispersion among patches. First, we prove that there is a unique global positive solution to the system with any initial positive value with probability 1, and we obtain the sufficient conditions that stochastically ensure the ultimate boundedness as well as the asymptotic polynomial growth of the population system. Next, we establish the sufficient conditions for the almost sure permanence in the mean, stochastic permanence, and extinction of the system. In particular, by constructing an appropriate integrating factor, we obtain the sufficient conditions for the almost sure weak permanence of the patch. The conditions obtained for permanence generalize the sufficient conditions established previously on the system without random environmental perturbations. Finally, we discuss the biological implications of the main results.

1. Introduction

Dispersal is a ubiquitous ecological phenomenon in the natural world, which has profound effects on both the permanence and evolution of species. In recent years, due to the ecological effects of local construction and development, such as the locations of manufacturing industries, the construction of dams and highways, as well as the development of tourism, complete habitats have been increasingly degraded into smaller patches, which may lead to the extinction of some rare species. Thus, the effect of dispersion on the possible persistence of endangered species has become an important subject in population biology.

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In many cases, some of these patches include suitable environments and richer food resources, whereas other patches contain little food. Species can disperse among patches with less food and patches with abundant food, but there is a possibility of species loss during dispersion among patches. In some of these patches, the species will become extinct without the contributions from other patches, and thus the species must live in a poor patchy environment. Many studies have investigated the time-dependent logistic population model with dispersal in poor patchy environments to obtain results related to permanence and extinction (see [9,10,7,8,6]).

In particular, Cui and Takeuchi [9] proposed the following system to describe the growth and dispersal of a time-dependent single-species system in poor patchy environments ($n \geq 2$)

$$\dot{x_i} = x_i[b_i(t) - a_i(t)x_i] + \sum_{j=1}^{n}(1-\lambda_{ij}(t))D_{ij}(t)x_j - \sum_{j=1}^{n} D_{ji}(t)x_i, \qquad i = 1, 2, \cdots, n, \tag{1.1}$$

where x_i $(i = 1, 2, \cdots, n)$ denotes the species x in patch i; $b_i(t)$, $a_i(t)$, $\lambda_{ij}(t)$, and $D_{ij}(t)$ are all continuous functions of time $t \in [0, +\infty)$, which are assumed to be periodic with common period $\omega > 0$; $b_i(t)$ is the intrinsic growth rate for species x in patch i; $a_i(t)$ represents the self-inhibition coefficient, which is assumed to be positive for $0 \leq t \leq \omega$; $\lambda_{ij}(t)$ expresses the loss of the species while moving from patch j to patch i; and $D_{ij}(t)$ is the dispersal coefficient of species x from patch j to patch i.

The model defined above is a deterministic model, where it is assumed that the parameters in the model are all deterministic irrespective of environmental fluctuations, but from a biological viewpoint, this imposes some limitations during the mathematical modeling of ecological systems. However, population systems in the real word are often inevitably affected by environmental noise, which is an important factor in an ecosystem (e.g., see Gard [12,13]). Therefore, it is useful to determine how noise affects population systems. There are various types of environmental noise. First, let us consider white noise, which arises from a nearly continuous series of small or moderate perturbations that have similar effects on the birth and death rates of all individuals (within each age or stage class) in a population [30]. Recall that $b_i(t)$ is the intrinsic growth rate for species x in patch i. In practice, we usually estimate this based on an average value plus an error that follows a normal distribution. If we still use $b_i(t)$ to denote the average growth rate, then the intrinsic growth rate becomes

$$b_i(t) \to b_i(t) + \sigma_i(t)\dot{B}_i(t),$$

where $\dot{B}_i(t)$ is white noise and $\sigma_i(t)$ is a continuous function of time $t \in [0, +\infty)$, which is assumed to be periodic with common period $\omega > 0$, thereby expressing the intensity of the white noise. As a result, Eq. (1.1) becomes the following single-species stochastic system in a patchy environment

$$\begin{aligned} dx_i = &\Big(x_i[b_i(t) - a_i(t)x_i] + \sum_{j=1}^{n}(1-\lambda_{ij}(t))D_{ij}(t)x_j - \sum_{j=1}^{n} D_{ji}(t)x_i\Big)dt \\ &+ \sigma_i(t)x_i dB_i(t), \qquad i = 1, 2, \cdots, n, \end{aligned} \tag{1.2}$$

where $B_i(t)$ is mutually independent Brownian motion with $B_i(0) = 0$ defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ that satisfies the usual conditions (i.e., it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Stochastic biological dynamics and stochastic epidemic models perturbed by white noise have been studied extensively, and we refer the reader to [29,20,34,26,35,36,33,21,11,15,37] and the references therein.

Furthermore, the population may be affected by sudden catastrophes, e.g., earthquakes, hurricanes, severe weather, floods, fire, or epidemics. However, stochastic dynamic system (1.2) is a pure diffusion-type