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Reduced-order finite difference extrapolation model based on proper orthogonal decomposition for two-dimensional shallow water equations including sediment concentration ☆

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ABSTRACT

In this study, we employ a proper orthogonal decomposition (POD) method to establish a POD-based reduced-order finite difference (FD) extrapolating model with very few degrees of freedom for two-dimensional shallow water equations that include the sediment concentration. We provide estimates of the error between the accurate solution and classical FD solutions, as well as those between the accurate solution and the POD-based reduced-order FD solutions. Moreover, we present two numerical simulation experiments to demonstrate that the POD-based reduced-order FD extrapolating model can greatly reduce the computational load. Thus, we validate both the feasibility and efficiency of the POD-based reduced-order FD extrapolating model.

1. Introduction

A system of shallow water equations (SWEs) can be used to describe the propagation and transformation of short waves in shallow waters, which are also referred to as the Saint-Venant system (see [13]). SWEs have extensive applications in ocean, environmental, hydraulic, and coastal engineering, such as open channel flows in rivers and reservoirs, tidal flows in estuaries and coastal water regions, bore wave propagation, and stationary hydraulic jumps and rivers, as mentioned in [16]. SWEs comprise a system of nonlinear partial differential equations (PDEs), so they generally have no analytical solutions, and thus we have to rely on numerical solutions.

Many previous studies have considered the numerical solutions for two-dimensional (2D) SWEs that only include the continuity equation and the momentum equations, i.e., that only include the water depth and

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the velocity of fluid, such as the finite volume (FV) method on unstructured triangular meshes proposed by Anatasiou and Chan in [1], the upwind methods described by Bermudez and Vazquez in [5], the parallel block preconditioning techniques given by Cai and Navon in [8], the optimal control technique with a finite element (FE) limited-area proposed by Chen and Navon in [11], the least-squares FE method of Liang and Hsu in [22], the finite difference (FD) Lax–Wendroff weighted essentially non-oscillatory (WENO) schemes proposed by Lu and Qiu in [25], the FE simulation technique of Navon in [39], the FD WENO schemes described by Qiu and Shu in [41], the Roe's approximate Riemann solver technique of Rogers et al. in [44], the essentially non-oscillatory and WENO schemes with an exact conservation property proposed by Vukovic and Sopta in [55], the explicit multi-conservation FD scheme of Wang in [56], the composite FV method on unstructured meshes given by Wang and Liu in [58], the high order FD WENO schemes of Xing and Shu in [61], the high order well-balanced FV WENO schemes and discontinuous Galerkin (DG) methods of Xing and Shu in [62], the positivity-preserving high order well-balanced DG methods of Xing et al. in [63], the dispersion-correction FD scheme of Yoon et al. in [66], the non-oscillatory FV method proposed by Yuan and Song in [67], the surface gradient method of Zhou et al. in [69], and the total variation diminishing FD scheme given by Wang et al. in [59]. However, the transport and sedimentation of silt and sand are important processes in changing natural environments, such as formation and evolution of deltas, the expansion of alluvial plains, and the migration of rivers. In addition, some serious problems need to be carefully considered in many hydraulic problems, such as irrigation systems, transportation channels, hydroelectric stations, ports, and other coastal engineering works. A model for 2D SWEs that includes the sediment concentration was established in [68] and some numerical methods have also been presented based on the optimal control approach (see [70]) and mixed FE technique (see [35,36]).

It is well known that the model based on classical FD scheme in [68] is the simplest and most convenient method for solving 2D SWEs including the sediment concentration, but it also contains many degrees of freedom (i.e., unknown quantities). Therefore, this method can cause many difficulties in real-life engineering computation, e.g., the accumulation of truncated errors in the computing process will increase very rapidly so the classical FD solutions may appear to deviate greatly after several computational steps. Therefore, it is extremely important to build a reduced-order FD scheme with sufficiently high accuracy and very few degrees of freedom in order to alleviate the accumulation of truncated errors and to reduce the computational load, as well as decreasing the time required to make the calculations and resource demands during the computational process, thereby obtaining more accurate simulations of the development of alluvial plains in an estuary and dam-break floods.

The proper orthogonal decomposition (POD) technique (see [18,20]) is one of the most effective means for reducing the degrees of freedom in numerical models of time-dependent complex and nonlinear problems. POD is based in statistics (see [14,24]) but it has been used widely in the study of coherent structures in turbulent flows (see [2,4,26,38,43,47]). In the past 30 years, the applications of the POD technique have developed greatly (for example, see [27,2,4,9,37,38,43,45–47]). In particular, over the past 20 years, it has been applied to the construction of numerical computational models for time-dependent PDEs, or reduced-basis models for parameterized PDEs, e.g., some Galerkin POD methods for a general equation in fluid dynamics proposed by Kunisch and Volkwein in [21], POD-based reduced-order FD schemes described by Luo's group in [31–34,50–52], POD-based reduced-order FE methods given by Luo et al. in [29,28,30], explicit reduced-order models for the stabilized FE approximation of the incompressible Navier–Stokes equations proposed by Baiges et al. in [3], an artificial viscosity POD given by Borggaard et al. in [6], an error estimation method for use in POD-based dynamic reduced-order thermal modeling of data centers described by Ghosh and Joshi in [15], the extrapolation-based acceleration of iterative solvers for application to simulations of 3D flows proposed by Grinberg and Karniadakis in [17], some iterative methods for model reduction by domain decomposition given by Buffoni et al. in [7], feedback control based on low-order modeling of the laminar flow past a bluff body by Weller et al. in [60], some reduced-basis models for parameterized PDEs given by Patera's group in [40,53,54,65] and by Rozza et al. in [42], a variational