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Well-posedness of 2-D and 3-D swimming models in incompressible fluids governed by Navier–Stokes equations [☆]

Alexander Khapalov [a,*,1], Piermarco Cannarsa [b], Fabio S. Priuli [c], Giuseppe Floridia [b]

[a] *Department of Mathematics, Washington State University, USA*
[b] *Department of Mathematics, University of Rome "Tor Vergata", Italy*
[c] *Istituto per le Applicazioni del Calcolo "M. Picone" (C.N.R.), Rome, Italy*

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ABSTRACT

We introduce and investigate the wellposedness of two models describing the self-propelled motion of a "small bio-mimetic swimmer" in the 2-D and 3-D incompressible fluids modeled by the Navier–Stokes equations. It is assumed that the swimmer's body consists of finitely many subsequently connected parts, identified with the fluid they occupy, linked by the rotational and elastic forces. The swimmer employs the change of its shape, inflicted by respective explicit internal forces, as the means for self-propulsion in a surrounding medium. Similar models were previously investigated in [15–19] where the fluid was modeled by the liner nonstationary Stokes equations. Such models are of interest in biological and engineering applications dealing with the study and design of propulsion systems in fluids and air.

1. Introduction

The swimming phenomenon has been the subject of interest for many researchers in various areas of natural sciences for a long time, aimed primarily at understanding biomechanics of swimming locomotion of biological organisms, see Gray [11] (1932), Gray and Hancock [12] (1951), Taylor [28] (1951), [29] (1952), Wu [32] (1971), Lighthill [22] (1975), and others. This research resulted in the derivation of a number of mathematical models for swimming motion in the (whole) $\mathbb{R}^2$- or $\mathbb{R}^3$-spaces with the swimmer to be used as the reference frame, see, e.g., Childress [5] (1981) and the references therein. In particular, based on the size of Reynolds number, it was suggested (for the purpose of simplification) to divide swimming models into three groups: microswimmers (such as flagella, spermatozoa, etc.) with "insignificant" inertia; "regular"

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* Corresponding author.
E-mail address: khapala@math.wsu.edu (A. Khapalov).
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swimmers (fish, dolphins, humans, etc.), whose motion takes into account both viscosity of fluid and inertia; and Euler's swimmers, in which case viscosity is to be "neglected".

It also appears that the following two, in fact, mutually excluding approaches were distinguished to model the swimming phenomenon (see, e.g., Childress [5] (1981)). One, which we can call the "*shape-transformation approach*", exploits the idea that the swimmer's shape transformations during the actual swimming process can be viewed as a set-valued map in time (see the seminal paper by Shapere and Wilczeck [26] (1989)). The respective models describe the swimmer's position in a fluid via the aforementioned maps, see, e.g., [13] (1981), [25] (2008), [6] (2011) and the references therein. Typically, such models consider these maps as *a priori prescribed*, in which case the question whether the respective maps are admissible, i.e., compatible with the principle of self-propulsion of swimming locomotion or not, remains unanswered. In other words, one cannot guarantee that the model at hand describes the respective motion as a self-propulsive, i.e., swimming process. To ensure the positive answer to this question one needs to be able to answer the question whether the a priori prescribed body changes of swimmer's shape can indeed be a result of actions of its internal forces under unknown in advance interaction with the surrounding medium.

The other modeling approach (we will call it the *"swimmer's internal forces approach" or SIF approach*) assumes that the available internal swimmer's forces are explicitly described in the model equations and, therefore, they determine the resulting swimming motion. In particular, these forces will define the respective swimmer's shape transformations in time as a result of an unknown-in-advance interaction of swimmer's body with the surrounding medium under the action of the aforementioned forces. For this approach, we refer to Peskin [23] (1975), Fauci and Peskin [7] (1988), Fauci [8] (1993), Peskin and McQueen [24] (1994), Tytell, Fauci et al. [31] (2010), Khapalov [17] and the references therein.

The original idea of Peskin's approach is to view a "narrow" swimmer as an immaterial "immersed boundary". Within this approach the swimming motion is defined at each moment of time by the explicit swimmer's internal forces. Following the ideas of Peskin's approach, Khapalov introduced the *immersed body SIF modeling approach* in which the bodies of "small" flexible swimmers are assumed to be identified with the fluid within their shapes, see [15–19] (2005–2014). Indeed, in the framework of Peskin's method the swimmer is modeled as an immaterial curve, identified with the fluid, further discretized for computational purposes on some grid as a collection of finitely many "cells", which in turn can be viewed as an immerse body, see Figs. 1 and 2. The idea here is to try, making use of mathematical simplifications of such approach, to focus on the issue of macro dynamics of a swimmer. The simplifications (they seem to us to be legitimate within the framework of our interest) include the reduction of the number of model equations and avoiding the analysis of microlevel interaction between a "solid" swimmer's body surface and fluid. It should be noted along these lines that in typical swimming models dealing with "solid" swimmers, the latter are modeled as "traveling holes" in system's space domain, that is, the aforementioned "microlevel" surface interaction is not in the picture as well.

In the above-cited works by Khapalov [15–19], the immerse body SIF approach was applied to the non-stationary Stokes equations in 2-D and 3-D dimensions with the goal to investigate the well-posedness of respective models and their controllability properties. In this paper our goal is to extend these results with respect to well-posedness to the case of Navier–Stokes equations in both the 2-D and 3-D incompressible fluids. To our knowledge, there were no previous publications investigating this issue within the SIF approach.

Related references on well-posedness of swimming models. To our knowledge, in the context of PDE approach to swimming modeling, the classical mathematical issues of well-posedness were addressed for the first time by Galdi [9] (1999), [10] (2002) for a model of swimming micromotions in $\mathbb{R}^3$ (with the swimmer as the reference frame). In [25] (2008) San Martin et al. discussed the well-posedness of a 2-D swimming model within the framework of the shape transformation approach for the fluid governed by the 2-D Navier–Stokes equations.